

136 Take-Home Exam

Due: **Tuesday, 11/10**

For this take home exam, you **may** consult your notes, worksheets, homeworks or your textbooks. You **may not** consult other books, the internet, or any person besides me. You **may not** use a calculator. You **may not** work together.

1. Graph the rational function

$$f(x) = \frac{x(x-6)}{(x-1)(x-5)}.$$

Label all x - and y -intercepts, and label all horizontal and vertical asymptotes. (Note: it's OK if your graph is somewhat imprecise between all those features, but the graph should be correct at or near those features. You probably will use (informal) ideas about limits.)

2. (a) Write an inequality to represent the statement “ m is within n of p ”.
- (b) Write an inequality to represent the statement “the computer predicted how many games the team would win with an error of no more than 2 games”.
3. (a) Find a quadratic inequality (see #6 from HW 3) whose solution set is $(-\infty, -1) \cup (4, \infty)$. Say a few words about how you came up with your inequality.
- (b) Find a function whose domain is $(-\infty, -1) \cup (4, \infty)$. Say a few words about how you know your function has this domain. (Piecewise defined functions don't count. If you get stumped, you can get most of the credit for a function whose domain is $(\infty, -1] \cup [4, \infty)$.)
4. (a) Find the roots of the quadratic function $y = x^2 + x - 6$ four different ways¹:
 - i. factoring.
 - ii. using the quadratic formula.
 - iii. completing the square and algebraically solving the appropriate quadratic equation.
 - iv. completing the square and viewing the graph as a transformation of $y = x^2$. (Be sure to explain how you can find the exact value of the roots using only what you know about $y = x^2$ and function transformations.)
- (b) Compare and contrast the above procedures. Discuss what each procedure does and does not accomplish, and also discuss any similarities or differences in the actual calculations that you did.

¹This version has been updated. You should tackle this quadratic and not the one on the older version...

5. Each of the following pages is a page from an undergraduate Algebra exam. Your job is to grade their performance.
- (a) Grade each problem on the following point scale: 0 - no work or nonsensical work that indicates no understanding of the problem; 1 - indicates some understanding of the problem, but with a crucial error, several smaller errors, or with several steps missing; 2 - on the right track, but with a minor error; 3 - totally correct. Justify each score that you give by citing evidence from the student's work.²
 - (b) Give complete solutions to problem (a) and problem (c) on the Algebra exam.

²Sorry, but I couldn't figure out a way to manipulate the image correctly... Please make sure I know which problem you are talking about, perhaps by giving each student a name....

1. (25 Points) Vital Errors:

(a) Simplify completely: $(a^{-2} + 3)^{-1}$

$$\frac{1}{\frac{1}{a^2} + \frac{3}{a^0}} = \frac{1}{1 + 3a}$$

Answer: $\frac{1}{1+3a}$

(b) Simplify completely: $\sqrt{4a^2b^4 + 16b^2a^4}$

$$2ab^2 + 4ba^2$$

Answer: $2ab^2 + 4ba^2$

(c) Simplify completely: $\frac{a^4b^3}{a^2b^5 - a^3b^3}$

$$\frac{a^2 + a^3}{b^2 - b^3} = \frac{a^5}{b^{-1}} = a^5b^1$$

Answer: a^5b^1

1. (25 Points) Vital Errors:

(a) Simplify completely: $(a^{-2} + 3)^{-1}$

$$a^2 + 3^{-1}$$

Answer: $a^2 + 3^{-1}$

(b) Simplify completely: $\sqrt{4a^2b^4 + 16b^2a^4}$

$$2ab^2 + 4ba^2$$

$$2ab(b + 2a)$$

Answer: $2ab(b + 2a)$

(c) Simplify completely: $\frac{a^4b^3}{a^2b^5 - a^3b^3}$

$$\frac{a^4b^3}{a^2b^5 - a^3b^3} = \frac{a^4b^3}{a^2b^3(b^2 - a)}$$

Answer: $\frac{a^4b^3}{a^2b^3(b^2 - a)}$

1. (25 Points) Vital Errors:

(a) Simplify completely: $(a^{-2} + 3)^{-1}$

$$= \frac{1}{a^{-2} + 3} \quad \frac{a^2}{3}$$

Answer: $\frac{a^2}{3}$

(b) Simplify completely: $\sqrt{4a^2b^4 + 16b^2a^4}$

$$\begin{aligned} & (2ab^2)^2 + 4(2ab^2)(2ab^2) + 4ba^2^2 \\ & 2ab^2 + 4ba^2 \\ & 2ab(b) + 2ab(2a) \qquad 2ab(b+2a) \end{aligned}$$

Answer: $2ab(b+2a)$

(c) Simplify completely: $\frac{a^4b^3}{a^2b^5 - a^3b^3}$

$$\begin{aligned} & \frac{ab^3(a^3b^3)}{b^2(a^2b^3) - a(a^2b^3)} \\ & \frac{ab^3(a^3b^3)}{a^2b^3 - a^3b^3} \end{aligned}$$

Answer: a^3b