

136 Homework #3

Due: **Tuesday, October 13**

This homework is in addition to the regularly assigned 171 weekly homework. Feel free to use your notes or book, and you also may work together so long as you write up your solutions on your own.

1. You can think broadly of multiplication as being the inverse of division. To make that more precise, you can think of the “multiply by 3 function”, $f(x) = 3x$, as the inverse of the “divide by 3 function”, $g(x) = \frac{x}{3}$. Assume that this relationship between multiplication and division is something important that you want to preserve. Explain why mathematicians might decide that division by zero shouldn’t be allowed.
2. In order to graph a parabola, our precalculus book tells you to complete the square, and then use information about a minimum or maximum value to find the vertex of the parabola, and then go from there.
 - (a) Use the book’s technique to graph the parabola $y = x^2 + 2x - 8$.
 - (b) Explain how to obtain the graph of $y = x^2 + 2x - 8$ by manipulating (translating, stretching or reflecting) the (hopefully) well-known graph of $y = x^2$. (Hint: look carefully at the equation you get when you complete the square.) Explain how, using this technique, you can find exact coordinates for the vertex of the parabola, as well as any roots that might exist (that is, don’t just estimate these coordinates).
3. Suppose you know that $x < y$. What can you conclude about the relationship between $x^2 + x - 12$ and $y^2 + y - 12$? Can you pick a restricted domain over which you can draw more conclusions about the relationship between $x^2 + x - 12$ and $y^2 + y - 12$? As always, be sure to justify your claims.
4. Some books define “completing the square” only in situations where you have quadratic polynomial on one side of an equation and 0 on the other. The process of “completing the square” then involves adding the appropriate number to both sides of the equation so that the polynomial becomes a perfect square polynomial.

$$\begin{aligned}0 &= x^2 + 6x + 7 \\2 + 0 &= (x^2 + 6x + 7) + 2 \\2 &= x^2 + 6x + 9 \\2 &= (x + 3)^2\end{aligned}$$

Some other books define “completing the square” in terms of manipulating just one side of a polynomial equation, as demonstrated below:

$$\begin{aligned}f(x) &= x^2 - 10x \\ &= x^2 - 10x + 25 - 25 \\ &= (x - 5)^2 - 25\end{aligned}$$

A student who only understands “completing the square” in the first sense got confused by the above example. The student was obstinate, and preferred to complete the square using the method they understood. Evaluate their work:

$$\begin{aligned}f(x) &= -2(x^2 - 20x) \\ f(x) + 100 &= -2(x^2 - 20x + 100) \\ f(x) + 100 &= -2(x - 10)^2 \\ f(x) &= -2(x - 10)^2 - 100\end{aligned}$$

5. Explain in your own words why the horizontal line test tells you whether a function is 1-to-1 or not.
6. Solve any way you like¹:

$$-2x^2 - x + 3 < 0$$

¹Well, solve in any way you can explain ... if you can explain how your calculator or computer gets its solution, then feel free to use them to solve this problem...