

136 Homework #4

Due: **Tuesday, 11/3**

This homework is in addition to the regularly assigned 171 weekly homework. Feel free to use your notes or book, and you also may work together so long as you write up your solutions on your own.

1. Suppose that you flip a coin once every second. Let $H(t)$ represent the total number of heads flipped by time t (measured in seconds), and let $T(t)$ represent the total number of tails flipped by time t .

(a) Find

$$\lim_{t \rightarrow \infty} H(t)$$

(b) Find

$$\lim_{t \rightarrow \infty} \frac{H(t)}{H(t) + T(t)}$$

Be sure to explain your answers.

2. For each of the following two items, I would like you to write a few lines or a short paragraph describing your thought process and answer to the question. I would like you to try to connect your ideas to the formal definition of the limit, but you can talk your way through it - your discussion doesn't need to be overly technical.

(a) For the function

$$f(x) = \begin{cases} 1 & x \text{ is rational} \\ 0 & x \text{ is irrational} \end{cases}$$

what is

$$\lim_{x \rightarrow 0} f(x)?$$

(b) For the function

$$g(x) = \begin{cases} x & x \text{ is rational} \\ 0 & x \text{ is irrational} \end{cases}$$

what is

$$\lim_{x \rightarrow 0} g(x)?$$

3. (a) Solve #16 from Section 2.3 of your calculus book. Include a sketch of the given situation.
(b) Explain what this calculation has to do with the formal definition of the limit. In addition, explicitly state, in limit notation, what limit this problem is trying to get at.
(c) Lastly, explain why the calculation you make in #16 is not sufficient to actually find the limit from part (b).
4. Suppose you know that $x < y$. What can you conclude about the relationship between $x^2 - 7x + 6$ and $y^2 - 7y + 6$? Can you pick a restricted domain over which you can draw more conclusions about the relationship between $x^2 - 7x + 6$ and $y^2 - 7y + 6$? As always, be sure to justify your claims.
5. Find an example of a limit or a limiting process from somewhere other than this course (maybe from other math or science courses or from “real life”). Be sure to clearly explain what the variables are, which variable is tending towards what value, and what the limit value actually is. Writing your example in (informal) limit notation should help you answer those questions - as an example

$$\lim_{n \rightarrow \infty} (\text{a regular } n\text{-sided polygon}) = \text{a circle.}$$

Keep in mind that just making an approximation is not necessarily the same as taking a limit. If there are ways in which you think your example might not technically be a limit, discuss those issues. Also, as always, you are encouraged to talk with your classmates about this problem, but I'd like everyone to come up with different examples.