Show all work. Explain your answers. You may use a dumb calculator, but one is not necessary. Answers which are fractions must be exact. One third must be written as $\frac{1}{3}$ and not rounded off to .33, for example.

Name_____________________________________

Circle the time of your TA section:

Tues 8:50    Tues 9:55    Thurs 8:50    Thurs 9:55

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<th>Problem</th>
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1. (20 %) A manufacturer of widgets has discovered that the number of defective widgets produced in her factory is a linear function of the total number of widgets which go thru the production line.

Suppose

2 defective widgets and 88 nondefective widgets were produced on Monday and

10 defective widgets and 120 nondefective widgets were produced on Tuesday.

On Wednesday a total of 160 widgets (both defective and nondefective) are to be produced. How many of these can she expect to be defective? (You may round your answer to the nearest whole number.)
2. (20 %)

\[\begin{align*}
3x + ay &= 6 \\
x - y &= 2
\end{align*}\]

a. If \(a = 1\), what is the solution of the above equation?

b. Show that the system above has a unique solution for all but one value of \(a\).

c. What is \(a\)?

d. For this \(a\) how many solutions does the system have?
3. (20 %) Find all solutions of the system:

\[
\begin{align*}
  y + z - 2w &= 0 \\
  x - z + w &= 1 \\
  x + y + z - 2w &= 1 \\
  2x + 2y + 2z - 2w &= 4
\end{align*}
\]
4. (20 %)

\[ A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & -1 \end{bmatrix} \quad C = \begin{bmatrix} -1 & -2 & 0 \\ -3 & 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} -1 & -2 \\ 1 & 3 \\ 0 & -1 \end{bmatrix} \]

(In the following problems you may say that “dimensions are wrong”.)

(a) What are the dimensions of \( D \)?

(b) \( 3C \)

(c) \( B + D \)

(d) \( AB \)

(e) \( DC \)
5. (20 %)

\[
A = \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 2 & 8 \\ 0 & 0 & 1 & 7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\]

(a) Find \( A^{-1} \).

(b) Find a matrix \( D \) such that \( DB = C \).
Answers

1. 16
2. a. \( x = 2 \) \( y = 0 \)
b. \( y = \frac{12}{(a + 3)} \) \( x = 2 + y \)
c. \( a = -3 \)
d. infinitely many
3. \( x = 1 \) \( y = 1 \) \( z = 1 \) \( w = 1 \)
4. a. \( 3 \times 2 \)
b. 
\[
3C = \begin{bmatrix} -3 & -6 & 0 \\ -9 & 3 & 0 \end{bmatrix}
\]
c. 
\[
B + D = \begin{bmatrix} 0 & -4 \\ 2 & 3 \\ 1 & -2 \end{bmatrix}
\]
d. 
\[
AB = \begin{bmatrix} 6 & -5 \\ 1 & 0 \\ 3 & -3 \end{bmatrix}
\]
e. 
\[
DC = \begin{bmatrix} 7 & 0 & 0 \\ -10 & 1 & 0 \\ 3 & -1 & 0 \end{bmatrix}
\]
5. a. 
\[
A^{-1} = \frac{1}{5} \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}
\]
b. 
\[
D = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\]