Show all work. Circle your answer.
No books, no notes, no calculator, no cell phones, no pagers, no electronic devices at all.
Solutions will be posted shortly after the exam: www.math.wisc.edu/~miller/m213

Name:______________________________

Circle your DIScussion section (column one):

TA: Youngsuk Lee

| DIS 301 | 8:50 T | 6322 SOC SCI |
| DIS 302 | 8:50 R | 215 INGRAHAM |
| DIS 303 | 9:55 T | 225 INGRAHAM |
| DIS 304 | 9:55 R | 495 VAN HISE |

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1. (5 pts) Find all first and second order partial derivatives of the function:

\[ f(x, y) = e^{x+y^2} \]

Circle your answer.
2. (5 pts) Find the average of the function

\[ f(x) = \frac{1}{x \ln(x)} \]

on the interval \([2, 5]\).

Circle your answer.
3. (6 pts) Graph the first octant of the plane determined by the equation:

\[ x + 3y + 2z = 6 \]

Circle your answer.
4. (8 pts) Determine whether the integral below converges or diverges and find its value if it converges.

\[ \int_{1}^{\infty} \frac{2x}{(x^2 + 1)^3} \, dx \]

Circle your answer.
5. (8 pts) The function

\[ f(x) = 2000e^{-0.01x} \]

represents a flow of money in dollars per year over 3 years. Assume 5% per year compounded continuously. Find

(a) the present value
(b) the accumulated amount after 3 years.

Circle your answer.
6. (10 pts) Find the critical points of the function and classify each as either saddle points or relative (or local) maximums or minimums.

\[ f(x, y) = 2x^2 - 4xy + y^4 + 2 \]

Circle your answer.
7. (8 pts) The graph of the function $y = \sqrt{x}$ for $x$ such that $0 \leq x \leq k$ is rotated around the $x$-axis, i.e. $y = 0$. The volume of the solid of rotation is $8\pi$. What is $k$?

Circle your answer.
Answers

1. \( f_x = e^x + y^2 = f_{xx}, \quad f_y = 2ye^x+y^2 = f_{yx}, \quad f_{yy} = 2e^x+y^2 + 4y^2e^x+y^2. \)

2. Substitute \( u = \ln(x) \) and \( du = \frac{1}{x} dx. \)

\[
\frac{1}{5 - 2} \int_{\frac{1}{2}}^{5} \frac{1}{x \ln(x)} \, dx = \frac{1}{3} (\ln(\ln(5)) - \ln(\ln(2)))
\]

3. The plane intersects the three axis at \((6, 0, 0)\) and \((0, 2, 0)\) and \((0, 0, 3)\)

4. Converges to \(\frac{1}{8}\). Substitute \( u = x^2 + 1 \) and \( du = 2x \, dx. \)

5. (a) \( \int_{0}^{3} e^{-0.05x}2000e^{-0.01x} \, dx = 2000 \int_{0}^{3} e^{-0.06x} \, dx = \frac{2000}{0.06} (1 - e^{-18}) \)

(b) \((e^{15})\) times part (a)

6. Saddle at \((0, 0)\), loc mins at \((1, 1)\) and \((-1, -1)\).

7. \[
\int_{0}^{k} \pi y^2 \, dx = \int_{0}^{k} \pi x \, dx = \pi \frac{k^2}{2} = 8\pi
\]
so \(k = 4\).