1.(5%) Draw the graph of a function \( f(x) \) such that \( f'(x) \) is positive on the intervals \((1, 2)\) and \((3, 4)\) negative on the interval \((2, 3)\).

2.(5%) Draw the graph of function \( f(x) \) such that \( f''(x) \) is is positive on the intervals \((1, 2)\) and \((3, 4)\) and negative on the interval \((2, 3)\).

3.(5%) Prove the special case of L’Hôpital’s Rule: Suppose that \( f(x) \) and \( g(x) \) are differentiable at \( a \), \( f(a) = g(a) = 0 \), and \( g'(a) \neq 0 \). Then \( \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)} \).

4.(5%) Draw a picture illustrating the Mean Value Theorem.

5.(5%) A light is hung 12 feet directly above a straight horizontal walk on which a girl 5 feet tall is walking. How fast is the girl’s shadow lengthening when she is walking away from the light at the rate of 168 feet per minute?

6.(5%) A woman has 320 yards of fencing for inclosing two separate lots, one of which is to square and other a rectangle three times as long as it is wide. Find the dimensions of each lot so that the total area inclosed shall be a minimum.

7.(5%) Prove the first part of the Fundamental Theorem of Calculus: Suppose \( f(x) \) is a continuous function on the interval \([a, b]\) and defining \( F(x) = \int_a^x f(t) \, dt \), then \( F'(x) = f(x) \).

8.(5%) Prove the second part of the Fundamental Theorem of Calculus: Suppose \( f(x) \) is a continuous function on the interval \([a, b]\) and \( G(x) \) is a function such that \( G'(x) = f(x) \), then \( \int_a^b f(x) \, dx = G(b) - G(a) \).

9.(5%) Find \( \frac{d}{dx} \int_0^{\sin(x)} \cos(t^4 + t^3) \, dt \).

10.(5%) Evaluate the definite integral \( \int_0^1 \frac{8x^3}{4x^2 + 1} \, dx \).

11.(5%) Suppose \( \int_2^3 3x^2 \tan(x^3) \, dx = \int_0^b \tan(u) \, du \). What are \( a \) and \( b? \)

12.(5%) Suppose \( \int_0^2 f(x) \, dx = 3 \) and \( \int_1^2 -4f(x) \, dx = 4 \). What is \( \int_0^1 f(x) \, dx? \)

13.(5%) Solve \( \frac{dw}{dx} = (\frac{x}{y})^3 \).

14.(5%) Solve \( \frac{dw}{dx} = \sin^2(x) \).

15.(5%) Solve \( \int x \tan^2(x^2 + 1) \, dx \).

16.(5%) Find \( \lim_{x \to \pi/2} \frac{1-\sin(x)}{1-\cos(4x)} \).

17.(5%) Standing on a 160 foot tower you throw a rock upwards at the velocity of 64 feet per second. What is the maximum height that rock achieves? (Recall that the acceleration due to gravity is \(-32 \) feet per second\(^2 \).)
18. (5%) Use the Trapezoid rule with \( n = 4 \) to approximate the definite integral \( \int_0^2 x^2 \, dx \).

For each of the following, circle True or circle False.

19. (1%) (True or False) \( \lim_{x \to 3} \frac{x^3 - 3}{x^2 - 3} = \lim_{x \to 3} \frac{1}{2x} \) by using L'Hôpital's Rule.

20. (1%) (True or False) If \( f(x) \) is a differentiable function, then it is integrable.

21. (1%) (True or False) If \( f(x) \) is a continuous function, then it is integrable.

22. (1%) (True or False) If \( f(x) \) is a function which is continuous on \([a, b]\), differentiable on \((a, b)\), and \( f(a) = f(b) \), then for some \( c \) in \((a, b)\) we have \( f'(c) = 0 \).

23. (1%) (True or False) \( \int_1^0 f(x) \, dx = -\int_0^1 f(x) \, dx \) for any continuous function \( f(x) \).

24. (1%) (True or False) \( \int_0^1 f(x) \, dx < 0 \) for any continuous function \( f(x) \).

25. (1%) (True or False) If \( f'(x) = g'(x) \) for all \( x \), then \( f(x) = g(x) \) for all \( x \).

26. (1%) (True or False) If \( f'(x) = 0 \) for all \( x \), then \( f(x) \) is a constant function.

27. (1%) (True or False) If \( f''(x) \) is positive on the interval \((a, b)\), then the function \( f'(x) \) is increasing on the interval \((a, b)\).

28. (1%) (True or False) If \( f'(x) \) is negative on the interval \((a, b)\), then the function \( f(x) \) is increasing on the interval \((a, b)\).
Answers Exam 2 Fall 91

5. 288
6. \( \frac{240}{7} \) by \( \frac{160}{7} \)
9. \( \cos(x) \cdot \cos(\sin^4(x) + \sin^3(x)) \)
10. \( \frac{1}{8} \)
11. \( a = 8 \) and \( b = 27 \)
12. 4
13. \( y = \frac{2}{7} + C \)
14. \( y = \frac{1}{2}x^3 - \frac{1}{4} \sin(2x) + C \)
15. \( \frac{1}{4}(\tan(x^2 + 1) - (x^2 + 1)) + C \)
16. \( \frac{1}{15} \)
17. 244
18. 2
19. False
20. True
21. True
22. True
23. True
24. False
25. False
26. True
27. True
28. False