Show all work. Use back of sheets if necessary.

Name ____________________________________________

Score
1. ____ (20)
2. ____ (20)
3. ____ (20)
4. ____ (20)
5. ____ (20)
6. ____ (20)
7. ____ (15)
8. ____ (15)
9. ____ (15)
10. ____ (10)
11. ____ (10)
12. ____ (15)

Total ____ (200) Grade ____

1. Find the general solution of the equation:

\[ \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = e^x \]
2. Find the general solution of the equation:

\[
\frac{d^4y}{dx^4} - 16y = 1
\]

3. Find \( \int_C x\,dx + 2xy\,dy + y^2\,dz \) where \( C \) is the curve \( C : [1, 2] \rightarrow \mathbb{R}^3 \) defined by \( c(t) = ti + t^2 j + \log(t)k \).
4. Find the maximum and minimum of \( f(x, y) = 2x + y \) subject to the constraint \( x^2 + 2y^2 = 2 \).

5. Find the volume of the pyramid bounded by the three coordinate planes and the plane \( x + 2y + 3z = 6 \).
6. Let \( f(X) = ||X||^4 \) where \( X \) is a vector in three dimensional space. Find the directional derivative of \( f \) at the point \((3, 4, 5)\) in either of the two directions along the curve of intersection of the two surfaces \( x^2 + y^2 = z^2 \) and \(-x^3 + y + z^2 = 2\).

7. Fill in the missing information on the second iterated integral.

\[
\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x, y) dy \, dx = \int \int f(x, y) \, dxdy
\]

\[
\int_{0}^{3} \int_{6-2x}^{6} f(x, y) \, dy \, dx = \int \int f(x, y) \, dxdy
\]

\[
\int_{-1}^{1} \int_{x^2}^{1} f(x, y) \, dy \, dx = \int \int f(x, y) \, dxdy
\]
In the remaining problems circle the best answer.

8. Given \( z = x^2 - xy + y^2 + x \) the point \((-2, -1)\) is:
   a. not a critical point
   b. a local maximum
   c. a local minimum
   d. a saddle point
   e. none of above

9. Let \( z = f(x, y) \), \( x = \cos(t) \), and \( y = \sin(2t) \). Given that \( \nabla f(0, 0) = (4, -3) \) the value of \( \frac{dz}{dt} \) when \( t = \frac{\pi}{2} \) must be:
   a. -10
   b. 2
   c. 6
   d. -4
   e. (1,0)
   f. (0,0)
   g. none of above

10. The general solution of the equation
    \[
    \frac{d^2y}{dx^2} - y = 0
    \]
    is:
    a. \( y = C_1x + C_2 \)
    b. \( y = C_1\cosh(x) + C_2\sinh(x) \)
    c. \( y = e^{C_1x} + e^{C_2x} \)
    d. all of above
    e. none of above
11. The general solution of the equation

\[ \frac{d^2y}{dx^2} + y = 0 \]

is

a. \( y = C_1 \cos(x) + C_2 \sin(x) \)
b. \( y = C_1 \sin(x + C_2) \)
c. \( y = C_1 \cos(x + C_2) \)
d. all of above

e. none of above

12. Match the curve with the given function

a. Roundoff error _____
b. Discretization error _____
c. Total error _____

13. Extra credit problem, do only if you have finished the exam. Put your answer on the back of this page. State the Fundamental Theorem of Calculus. State the Fundamental Theorem of Algebra. Prove either one.
Answers

1. \( y = C_1 e^x + C_2 x e^x + \frac{1}{2} x^2 e^x \)
2. \(-\frac{1}{16} + C_1 e^{2x} + C_2 e^{-2x} + C_3 \sin(2x) + C_4 \cos(2x)\)
3. \(\frac{601}{20}\)
4. 3, -3
5. 6
6. The gradient of \( f \) is 200(3, 4, 5). A tangent vector to the curve is (45, 105, 111). The directional derivative is \( \pm \frac{74000}{\sqrt{2819}} \).
7. \( \int_0^6 \int_{1/2(6-y)}^3 f(x, y) \, dx \, dy, \int_0^2 \int_0^{2\pi} r^3 \, d\theta \, dr, \int_1^0 \int_{-\sqrt{x}}^{\sqrt{x}} f(x, y) \, dx \, dy \)
8. a
9. b
10. b
11. d
12. This had some hand drawn curves. The roundoff error increases as the step size decreases and decreases as the step size increases. The discretization error is just the opposite. Therefore the total error (as a function of step size) looks like a U shaped curve.