1. A point is chosen at random on a line segment of length $L$ and the line segment is then cut at that point into two subsegments. Find the probability that the ratio of the shorter to the longer subsegment is less than $\frac{1}{3}$.

2. In the following use the normal approximation to the binomial distribution (DeMoivre-Laplace). You may express your answer in terms of $\Phi$. One thousand rolls of a fair die will be made. If the number 2 appears exactly 500 times, compute the approximate probability that the number 5 appears between 90 and 110 times.

3. If $X$ is a random variable with the exponential distribution with parameter $\lambda = 1$. Suppose $Y = e^X$. Find the probability that $2 < Y < 3$.

4. Suppose two balls are randomly selected without replacement from an urn containing 2 white and 3 red balls. Let $W$ be the number of white balls selected and $R$ be the number of red balls selected. Give the joint probability mass function of $W$ and $R$.

5. Three points $X, Y, Z$ are randomly selected (uniformly and independently) in the unit interval $[0, 1]$. What is the probability that $X < YZ$?

6. An ordinary deck of playing cards is shuffled and then seven cards are dealt out one at a time. Let $A$ be the number of aces dealt out. Find the expected value of $A$ and the variance of $A$.

7. You deal out a well-shuffled pack of cards until you have seen five spades. How many cards can you expect to deal out?

8. A fair die is rolled until we roll a six. Let $S$ be the sum of the rolls. For example, if we roll 4, 1, 4, 2, 6, then $S = 4 + 1 + 4 + 2 + 6 = 17$. Find the expected value of $S$. 
Answers

1. \[ \frac{1}{2} \]
2. \[ \Phi \left( \frac{10}{\sqrt{8}} \right) - \Phi \left( -\frac{10}{\sqrt{8}} \right) \approx 0.7 \]
3. \[ \frac{1}{6} \]
4. 
   \[ P(W = 2, R = 0) = \frac{1}{10} \]
   \[ P(W = 1, R = 1) = \frac{6}{10} \]
   \[ P(W = 0, R = 2) = \frac{3}{10} \]
   
   all other combinations have probability 0.
5. \[ \frac{1}{3} \]
6. \[ E(A) = \frac{7}{13} \approx 0.5385 \]
   \[ var(A) = 7\left(\frac{1}{13}\right)^2 + 42\left(\frac{1}{13}\right)\left(\frac{3}{13}\right) - \frac{1}{13} = \frac{1}{13} \approx 0.4386 \]
7. \[ 5 + 39\left(\frac{2}{13}\right) \approx 18.93 \]
8. 21