1. Define $\Pi_{i \in I} X_i = \{ \langle x_i : i \in I \rangle : \forall i \in I \ x_i \in X_i \}$

The $I$-sequence $\langle x_i : i \in I \rangle$ which is also written $(x_i)_{i \in I}$, is a function with domain $I$ which maps each $i \in I$ to an element $x_i \in X_i$.

2. If $X_i$ for $i \in I$ are topological spaces, then what is a basic open subset of $\Pi_{i \in I} X_i$ (in the Tychonoff product topology)?

A set of the form $\Pi_{i \in I} U_i$ where $U_i \subseteq X_i$ is open for every $i \in I$ and $U_i = X_i$ for all but finitely many $i \in I$.

3. If $X_i$ for $i \in I$ are topological spaces, then what is a basic open subset of $\square_{i \in I} X_i$ (in the Box product topology)?

A set of the form $\Pi_{i \in I} U_i$ where $U_i \subseteq X_i$ is open for every $i \in I$.

4. A metric $d$ on $X$ is a function $d : X \times X \to [0, \infty)$ such that
   
   (a) for all $x, y \in X \ d(x, y) = 0$ iff $x = y$,
   
   (b) for all $x, y \in X \ d(x, y) = d(y, x)$, and
   
   (c) for all $x, y, z \in X \ d(x, z) \leq d(x, y) + d(y, z)$

5. If $d$ is a metric on $X$ and $\epsilon > 0$, then $B_d^\epsilon(x) = \{ y \in X : d(x, y) < \epsilon \}$

6. If $(X, d)$ is a metric space then $U \subseteq X$ is open
   
   iff $\forall x \in U \ \exists \epsilon > 0 \ B_d^\epsilon(x) \subseteq U$

7. For a sequence $\langle x_n : n \in \omega \rangle$ in a space $X$ then $\lim_{n \to \infty} x_n = x_0$ iff for all $U$ an open neighborhood of $x_0$ for all but finitely many $x_n \in U$.

8. For $X$ a topological space, $A \subseteq X$ is clopen iff $A$ is closed and open.

9. For $X$ a topological space, $X$ is connected iff it contains no nontrivial clopen set. ($X$ and the empty set are the trivial clopen sets.)

10. $U$ is an open cover of $X$ iff the elements of $U$ are open and $\bigcup U = X$.

11. $V$ is a subcover of the cover $U$ iff $V \subseteq U$ and $\bigcup V = X$.

12. A topological space $X$ is compact iff every open cover of $X$ has a finite subcover.