

Open games are determined (Gale-Stewart [11]).

G_δ games are determined (Wolfe [29])

P implies ω_1 is inaccessible in L (Levy [15])

AD implies P, every set of reals is countable or contains a perfect subset (Davis [7])

Inconsistent versions of AD (Mycielski [20])

DC iff Baire category theorem (Blair [5], Goldblatt [12])

(ZFC) $L[\mathbb{R}] \models \text{“ZF+DC”}$ (Takeuti [26])

Equivalence of $AD_{\mathbb{R}}$ and AD for games of length ω^2 (Blass [6])

Banach-Mazur games and Baire property, AD implies BP (see Oxtoby [22])

$AD_{\mathbb{R}}$ implies every set $A \subseteq [\omega]^\omega$ is completely Ramsey (Prikry [23] also Ellentuck [9])

AD for Banach games is equivalent to AD (Freiling [10], Becker [4])

AD implies LM, every set is Lebesgue measurable (Mycielski, Swierczkowski [21])

The boundedness theorem for $\mathbf{\Pi}_1^1$

AD implies the club filter on ω_1 is an ultrafilter, (Solovay 1967, see Kechris [14])

Wadge's Lemma, well-foundedness of Wadge ordering, (Wadge, Martin, Monk, see Van Wesep [27])

$PWO(\Gamma)$ implies $Red(\Gamma)$ implies $Sep(dual\Gamma)$ and not $Sep(\Gamma)$

$PWO(\mathbf{\Pi}_1^1)$, $PWO(\mathbf{\Sigma}_2^1)$

(V=L) $PWO(\mathbf{\Sigma}_n^1)$ for $n \geq 2$ (Addison [1],[2] see also [19])

Projective Determinacy PD implies $PWO(\mathbf{\Sigma}_n^1)$ for n even and $PWO(\mathbf{\Pi}_n^1)$ for n odd, (Blackwell, Martin, Addison, Moschovakis see [3], [19])

The Separation holds on exactly one side of a nonselfdual Wadge class, (Steel [25], Van Wesep [28])

Borel sets are determined, (Martin [17],[18])

Existence of measurable cardinal equivalent to an elementary embedding (Los). Measurable implies $V \neq L$ (Scott).

Existence of measurables does not decide continuum hypothesis (Levy, Solovay). No nontrivial embedding of V into V (Kunen).

Measurables are Ramsey, limit of Ramsey's (Rowbottom). Erdos cardinals are strongly inaccessible.

Analytic sets are determined assuming there is a measurable cardinal (or even Erdos cardinal), (Martin [16])

The theory of 0^\sharp , Club of indiscernibles for L , Π_2^1 -singleton (Silver, Solovay, Rowbottom).

Existence of 0^\sharp equivalent to a nontrivial elementary embedding of L into L (Kunen).

Sharps are equivalent to Analytic Determinacy, (Harrington [13] see also Sami [24])

Equivalence of sharps, Wadge's lemma for Π_1^1 , Borel isomorphism, and $< \omega^2 - \Pi_1^1$ -det (Harrington, Steel, Martin, see also Dubose [8])

References

- [1] Addison, J. W., Some consequences of the axiom of constructibility. *Fund. Math.* 46 1959 337–357.
- [2] Addison, J. W., Separation principles in the hierarchies of classical and effective descriptive set theory. *Fund. Math.* 46 1959 123–135.
- [3] Addison, J. W.; Moschovakis, Yiannis N., Some consequences of the axiom of definable determinateness. *Proc. Nat. Acad. Sci. U.S.A.* 59 1968 708–712.
- [4] Becker, Howard, Determinacy of Banach games. *J. Symbolic Logic* 50 (1985), no. 1, 110–122.
- [5] Blair, Charles E., The Baire category theorem implies the principle of dependent choices. *Bull. Acad. Polon. Sci. Sr. Sci. Math. Astronom. Phys.* 25 (1977), no. 10, 933–934.
- [6] Blass, Andreas, Equivalence of two strong forms of determinacy. *Proc. Amer. Math. Soc.* 52 (1975), 373–376.
- [7] Davis, Morton, Infinite games of perfect information. 1964 *Advances in game theory* 85–101 Princeton Univ. Press, Princeton, N.J.

- [8] Dubose, Derrick Albert, The equivalence of determinacy and iterated sharps. *J. Symbolic Logic* 55 (1990), 502–525.
- [9] Ellentuck, Erik, A new proof that analytic sets are Ramsey. *J. Symbolic Logic* 39 (1974), 163–165.
- [10] Freiling, Chris, Banach games. *J. Symbolic Logic* 49 (1984), no. 2, 343–375.
- [11] Gale, David; Stewart, F. M. Infinite games with perfect information, *Contributions to the theory of games*, vol. 2, pp. 245–266, *Annals of Mathematics Studies*, no. 28, Princeton University Press, Princeton, N. J., 1953.
- [12] Goldblatt, Robert, On the role of the Baire category theorem and dependent choice in the foundations of logic. *J. Symbolic Logic* 50 (1985), no. 2, 412–422.
- [13] Harrington, Leo, Analytic determinacy and 0^\sharp . *J. Symbolic Logic* 43 (1978), no. 4, 685–693.
- [14] Kechris, Alexander S., AD and projective ordinals. *Cabal Seminar 76–77 (Proc. Caltech-UCLA Logic Sem., 1976–77)*, pp. 91–132, *Lecture Notes in Math.*, 689, Springer, Berlin, 1978.
- [15] Levy, Azriel, Definability in axiomatic set theory. II. *1970 Mathematical Logic and Foundations of Set Theory (Proc. Internat. Colloq., Jerusalem, 1968)* pp. 129–145 North-Holland, Amsterdam
- [16] Martin, Donald A., Measurable cardinals and analytic games. *Fund. Math.* 66 1969/1970 287–291.
- [17] Martin, Donald A., Borel determinacy. *Ann. of Math. (2)* 102 (1975), no. 2, 363–371.
- [18] Martin, Donald A., A purely inductive proof of Borel determinacy. *Recursion theory (Ithaca, N.Y., 1982)*, 303–308, *Proc. Sympos. Pure Math.*, 42, Amer. Math. Soc., Providence, R.I., 1985.
- [19] Yiannis N. Moschovakis, **Descriptive set theory**, Elsevier-North Holland, 1980.
- [20] Mycielski, Jan, On the axiom of determinateness. *Fund. Math.* 53 1963/1964 205–224.
- [21] Mycielski, Jan; Świerczkowski, S., On the Lebesgue measurability and the axiom of determinateness. *Fund. Math.* 54 1964 67–71.
- [22] J.Oxtoby, **Measure and category**, Springer-Verlag, 1971.
- [23] Prikry, Karel, Determinateness and partitions. *Proc. Amer. Math. Soc.* 54 (1976), 303–306.

- [24] Sami, Ramez L., Analytic determinacy and 0^\sharp . A forcing-free proof of Harrington's theorem. *Fund. Math.* 160 (1999), 153–159.
- [25] Steel, John R., Determinateness and the separation property. *J. Symbolic Logic* 46 (1981), no. 1, 41–44.
- [26] Takeuti, Gaisi, Hypotheses on power set. *Axiomatic set theory* (Proc. Sympos. Pure Math., Vol. XIII, Part I, Univ. California, Los Angeles, Calif., 1967), pp. 439–446. Amer. Math. Soc., Providence, R.I., 1971.
- [27] Van Wesep, Robert, Wadge degrees and descriptive set theory. *Cabal Seminar 76–77* (Proc. Caltech-UCLA Logic Sem., 1976–77), pp. 151–170, *Lecture Notes in Math.*, 689, Springer, Berlin, 1978.
- [28] Van Wesep, Robert A., Separation principles and the axiom of determinateness. *J. Symbolic Logic* 43 (1978), no. 1, 77–81.
- [29] Wolfe, Philip, The strict determinateness of certain infinite games. *Pacific J. Math.* 5 (1955), 841–847.