Function Graphs

The Real Number Line

intuitively:

- 0 is an arbitrary point on the line
- 1 is a point to the right of 0
- 2 is a point to the right of 1 and twice the distance from 0 as 1, etc. $\mathbb{N}, \mathbb{Z}$
- we fill in the rational numbers $\mathbb{Q}$.
- and then the irrational numbers $\mathbb{R} \setminus \mathbb{Q}$ via:

  Dedekind Cut or Cantors approach via equivalence classes of convergent sequences of rational numbers

Intervals

Suppose a, b $\in \mathbb{R}$, $a < b$ means that a lies to the left of b on the real number line.

the open interval

$$(a,b) = \{x \in \mathbb{R} \mid a < x < b\}$$

the closed interval

$$[a,b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$$

$\mathbb{R}^2$ and function graphs

The coordinate plane $\mathbb{R}^2$

$$\mathbb{R}^2 = \{(x,y) \mid x \in \mathbb{R},\ y \in \mathbb{R}\}$$

The graph of the function f:

$$G_f = \{(x,y) \in \mathbb{R} \mathrm{x} \mathbb{R} \mid y = f(x)\}$$

Function Graphs

- **Lines**: $f(x) = mx + b$

Example: graph the set of points in $\mathbb{R}^2$ satisfying $|x+y| < 1$.

- **Polynomials**:

are generated by f(x) = x^n, $n \in \mathbb{N}$.

- **Rational Functions**:

have the form f(x) = $p(x)/q(x)$,

where *p(x), q(x)* are polynomials.

Example: Graph f(x) = $1/x - a$, $a \in \mathbb{R}$.

- transcendental functions:

trigonometric functions, exponentials, logarithms.

Exercise

Graph these functions:

1. $f(x) = \sin x$
2. $f(x) = \sin 1/x$
3. $f(x) = x \sin 1/x$
4. $f(x) = x^2 \sin 1/x$

these functions are crucial to understanding analysis.

Not a function graph

The **unit circle**: $\{(x,y) \in \mathbb{R}^2 \,|\, x^2 + y^2 = 1\}$

It is made up of the two branches of the square root function:

$$y_0 = \sqrt{1-x2}\,, \qquad |x| \leq 1$$

$$y_1 = -\sqrt{1-x2} \qquad |x| \leq 1$$

overlap at $x = \pm 1$.

Exercise

Describe the graph of g in terms of the graph of f for

- $g(x) = f(x) + c$
- $g(x) = f(x+c)$
- $g(x) = c \cdot f(x)$
- $g(x) = f(cx)$
- $g(x) = f(1/x)$

Exercise (continued)

Describe the graph of g in terms of the graph of f for

- $g(x) = |f(x)|$
- $g(x) = \max(f, 0)$
- $g(x) = \min(f, 0)$
- $g(x) = \max(f, 1)$