(1) Milne 7-1
(2) Milne 7-2
(3) Fill in the details in the section "Class numbers of cyclotomic fields" section of Milne (p. 101) to prove that $\mathbb{Q}(\zeta_{23})$ has class number $> 1$. For what other cyclotomic fields can you make a similar argument?
(4) Let $K = \mathbb{Q}(\zeta_8)$ and $\wp = (1 - \zeta_8)$. Show that $Nm\wp = 2$ and that complex conjugation acts trivially on $\mathcal{O}_K/\wp^2$. Find a fundamental unit in $K$. (Hint: first find a fundamental unit in $\mathbb{Q}(\zeta_8) \cap \mathbb{R}$ and look at Proposition 5.12.)
(5) Show that there are no integer solutions to $x^2 - 82y^2 = \pm 2$. You may use sage.
(6) Use sage to list the regular primes $< 100$.
(7) Use sage to compute how many of the first 1000 primes are regular.