MATH 748: HOMEWORK 7

(1) Find the class numbers of \( \mathbb{Q}(\sqrt{-163}) \), \( \mathbb{Q}(\sqrt{-10}) \), and \( \mathbb{Q}(\sqrt{14}) \). Make sure to prove any claims that ideals are non principal. (Hint: you can use Dedekind’s theorem to help tell you how primes factor in \( \mathcal{O}_K \).)

(2) Milne 4-4

(3) Milne 4-5

(4) Milne 4-6

(5) Use sage to compute the \( e_i, f_i \) of the primes occurring in the factorization of \( p\mathcal{O}_K \) for the first 200 primes \( p \) in \( \mathbb{Z} \) for 4 different quadratic fields \( K \), 4 different Galois cubic fields \( K \), and 4 different non-Galois cubic fields \( K \). For each \( K \), make a table tabulating how many of each splitting type (i.e. \( (e_i, f_i) \) data) occur. Notice anything about how often each possibility occurs? What is needed to prove something about how often each possibility occurs in the quadratic case?