Math 130
Final Review

1 Topic Outline:
The final exam will be cumulative, but the new material will be emphasized. This outline covers primarily the new material. Of the 8 questions on the exam, about 4 will be based on the material in this outline, and about 4 will be based on material from the midterms. Any topic that we’ve discussed in class so far may appear on the exam.

1. Ratios and Proportions
   (a) Definitions
   (b) Equivalent Ratios
   (c) Solving word problems (and writing teacher’s solutions)

2. Percents
   (a) Definitions
   (b) Solving problems with the “unitary method”

3. Rates
   (a) Definitions
   (b) Rate conversions
   (c) Solving “simultaneous work problems”
   (d) Solving “average work rate problems”

4. Negative Numbers and Integers
   (a) Vector, chip, and money models
5. Decimals, Rationals, and Real Numbers

(a) Definitions
(b) Extending the usual arithmetic algorithms to decimals
(c) Know when the decimal expansion of a fraction will terminate or repeat
(d) Convert fractions to decimals (including the shortcut for terminating decimals)
(e) Convert decimals to fractions
(f) Basic facts about irrational numbers
(g) Know how to identify if a real number is rational or irrational

2 Sample Problems

The following problems are intended to help you test your knowledge of the major topics on the exam. They do not match the actual questions on the exam (although I hope that anyone who is comfortable solving these problems will also do well on the exam). These questions are organized roughly in the order that the topics appear in the above list.

Problem 1. In a bag of marbles, the ratio of red marbles to blue marbles is 4:3 and the ratio of blue marbles to white marbles is 6:7. What is the ratio of white marbles to red marbles? What is the smallest number of marbles that can be in a bag satisfying the conditions above?

Problem 2. Write a teacher’s solution to the following word problem:

Alice has \( \frac{7}{11} \) as many stamps as Bob. If Alice gives Bob \( \frac{2}{21} \) of her stamps, what will be the ratio of the number of Bob’s stamps to Alice’s?

Problem 3. Store A sells office chairs for 90% of what store B sells them for. If store A has a 20%-off sale and store B has a 30%-off sale on the same day, what percent of store B’s sale price will store A’s sale price be? If store B ordinarily sells the office chairs for $100 each, which store should you shop at on the day of the sale, and how much will you save (vs. the other store on that day).
(Note: the correct answer to the first part of this problem is not an integer percent, so make sure you are able to compute this answer to the nearest tenth of a percent.)

**Problem 4.** Alice can paint a fence in 10 hours and Bob can paint the same fence in 6 hours. If Alice, Bob, and Carl paint the fence together, it only takes them 3 hours. How long would it take Carl to paint the fence alone? How long would it take Bob and Carl to paint the fence?

**Problem 5.** I walk up Bascom Hill at 3mph, and I walk down Bascom hill at 5mph. If I walk up Bascom hill and back down, what was my average speed for this trip? (Note: The answer is not 4mph. Give a decimal answer.)

**Problem 6.** Simplify the following into a fraction with all exponents positive:
\[
\frac{36^4 \cdot 6^{-4} \cdot 96^{-3}}{48^{-2} \cdot 15^2 \cdot 256}
\]

**Problem 7.** Prove the following:

1. \(-(-a) = a\) for any integer \(a\).

2. If \(a\) and \(b\) are integers with \(a < b\), then \(c - b < c - a\) for any integer \(c\).

**Problem 8.** Consider the following numbers:

1. 3
2. 0
3. -7
4. 0.934
5. \(0.\overline{93}\)
6. \(0.34\overline{12}\)
7. \(\frac{4}{5}\)
8. \(\sqrt{2}\)
9. \(\sqrt{36}\)
10. $0.20200200020002\ldots$

11. $\pi$

Which of the numbers above are whole numbers? Integers? Rational numbers? Irrational numbers? Real numbers?

**Problem 9.** 1. Convert the following decimals to fractions (in lowest terms):

   (a) $0.238$
   (b) $6.35$
   (c) $0.\overline{279}$
   (d) $0.2\overline{314}$

2. Convert the following fractions to decimals (make sure you can convert the terminating ones without long division):

   (a) $\frac{67}{80}$
   (b) $\frac{36}{75}$
   (c) $\frac{6}{11}$
   (d) $\frac{17}{7}$

**Problem 10.** Write down 5 different irrational numbers. Try to only use a square root for one of your examples.