Homework #4, Algebraic Reasoning; Patterns & Rules

1. A sequence is called **arithmetic** if the difference between successive terms is a constant. A sequence is called **geometric** if the ratio between successive terms is constant. Fill in the following patterns and write both closed and recursive rules the \( n^{th} \) number. Then decide whether the pattern is arithmetic or geometric, and justify using the definitions.

(a) 3, 9, __, 81, __, ...
(b) -4, -1, 2, __, 8, __, ...
(c) 9, 16, __, 30, 37, __, ...
(d) 4, 8, __, 32, __, ...
(e) Now, explain how you can see the difference between arithmetic and geometric sequences in your closed rules and in your recursive rules.

2. The following sequences are neither arithmetic nor geometric. For each pattern, prove this (you’ll need to refer to the definitions in question 1) and then write a rule (of either kind).

(a) 1, 1, 2, 3, 5, __, 13, __, ...
(b) 0, 2, 6, 12, __, 30, __, ...

3. *** A scientist is studying some bacteria using a microscope.

(a) She first places three bacteria onto the slide. She then observes that, at the end of each minute, every bacterium splits into two new bacteria. Write both closed and recursive rules that describe the bacteria count \( b(t) \) after \( t \) minutes. Then show how to use each of your rules to find how many minutes must pass from the time she first placed the three bacteria for there to be over 100 bacteria. (Note: You do not have to solve your equation. Instead, give the first whole minute for which there will be over 100 bacteria.)

(b) Next, she moves to a different microscope to check on the growth of another organism. She counts that there are currently 65 organisms on the slide. Her lab mate has determined that this organism reproduces so that the number of organisms after \( t \) minutes is given by \( r_{t+1} = 2r_t - 1 \). If five minutes have passed since the organism started to reproduce, how many of the organism were there at the start?

4. (a) Write a function to describe the rule for the numbers in this table. Give another pair of numbers that fit.

<table>
<thead>
<tr>
<th></th>
<th>( a_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>10</td>
<td>23</td>
</tr>
<tr>
<td>50</td>
<td>103</td>
</tr>
</tbody>
</table>
(b) Plot these points on a graph (as ordered pairs), being sure to label. Now compare the table, rule, and graph. What are some positives and negatives of each representation of the pattern?

5. Recall this pattern that we saw in class:

(a) Find a closed form rule for the perimeter of the figure in the $n^{th}$ stage (remember, perimeter only counts the outside edges). Define all symbols.

(b) Now, it takes 4 edges to draw the first figure, 16 edges to draw the second, and so on. Find a closed form rule for the number of edges needed for the figure in the $n^{th}$ stage.

6. Again refer to the pattern above.

(a) Write another closed form rule for the perimeter of the figure, but this time, suppose that you are given the number of blocks in the figure rather than the stage number $n$. For example, your rule should read something like “the perimeter is 7 times the number of blocks plus 11,” though that is not the right answer, of course.

(b) As above, write a closed rule for the number of edges using the number of blocks.

(c) Recall that, in class, we found the number of blocks in the figure in the $n^{th}$ stage to be $4n - 3$. Show how you can use this rule along with the rules you wrote in (a) and (b) of this problem to come up with the rules you found in problem 5.