Assignment #1, due Tuesday, September 9

Problem I  In the following, let $V$ be a vector space, let $x \in V$ be a vector, and let $\alpha$ be a scalar. Let $\vec{0}$ denote the identity element of $V$, while $0$ denotes the scalar zero. Using the axioms for a vector space, prove the following statements. Use complete sentences when writing up your proof.

1. $0 \cdot x = \vec{0}$.
2. $\alpha \cdot \vec{0} = \vec{0}$.
3. If $\alpha \cdot x = \vec{0}$ then either $\alpha = 0$ or $x = \vec{0}$.
   (Note that in mathematics, “either – or –” means either one or the other or both).

Problem II  Let $P_4$ denote the vector space of all real polynomials of degree less than or equal to 4. Let $W_0$ denote the set of all polynomials $P \in P_4$ such that $x \cdot P'(x) = P(x)$.

Let $W_1$ denote the set of all polynomials $P \in P_4$ such that $x \cdot P'(x) = P(x) + x^2$.

1. If $P(x) = a x^4 + b x^3 + c x^2 + d x + e$, what are the conditions on the 5-tuple $(a, b, c, d, e)$ so that $P \in W_0$? What if we want $P \in W_1$?
2. Show that $W_0$ is a subspace of $P_4$ and that $W_1$ is not a subspace of $P_4$.

Problem III  Let $V$ denote the set of strictly positive real numbers. (This is often denoted by $\mathbb{R}^+$.). Define a new “sum” of two elements $x, y \in V$ to be their product $x \cdot y$. Also if $\alpha \in \mathbb{R}$ is a scalar and $x \in V$, define a new “scalar multiplication” of $\alpha$ times $x$ to be the number $x^\alpha = e^{\alpha \ln(x)}$. Prove that with these new definitions, $V$ is a vector space over $\mathbb{R}$.

Problem IV  We can regard the complex numbers $\mathbb{C}$ as a complex vector space with complex numbers as scalars, or as a real vector space with real numbers as scalars.

1. If we think of $\mathbb{C}$ as a complex vector space, is the complex number $i = \sqrt{-1}$ a scalar multiple of the number 1? Which complex numbers are scalar multiples of 1?
2. If we think of $\mathbb{C}$ as a real vector space, is the complex number $i = \sqrt{-1}$ a scalar multiple of the number 1? Which complex numbers are scalar multiples of 1?

In addition, do the following problems:

Section 1.5:  # 4, 5, 10, 11, 25, 26;

Section 1.10:  # 3, 7, 8, 11, 14, 15 (You do not yet have to compute the dimension).