Assignment #7, Due Wednesday November 26

Read chapter 8, sections 6, 10, 12, 13, 15, 19, 20, and 21 in the text. Do the following problems:

**Problem I**

Let \( \bar{x} = (x_1, \ldots, x_n) \) and \( \bar{y} = (y_1, \ldots, x_n) \) be points in \( \mathbb{R}^n \).

(a) Show that if \( ||\bar{x} - \bar{y}|| < \delta \), then \( |x_j - y_j| < \delta \) for \( 1 \leq j \leq n \).

(b) If \( |x_j - y_j| < \delta \) for \( 1 \leq j \leq n \), what can you say about \( ||\bar{x} - \bar{y}|| \)?

**Problem II**

Using the definition of limits, prove the following:

(a) Let \( f : \mathbb{R} \to \mathbb{R} \) and suppose that \( f \) is differentiable at \( a \in \mathbb{R} \). Prove that \( f \) is continuous at \( a \).

(b) Let \( F : \mathbb{R}^n \to \mathbb{R}^n \) and suppose that \( F \) is differentiable at \( \bar{a} \in \mathbb{R}^n \). Prove that \( F \) is continuous at \( \bar{a} \).

**Problem III**

Let \( u(x, y) = x^2 - y^2 \) and \( v(x, y) = 2xy \).

(a) Sketch the level sets \( \{ (x, y) \in \mathbb{R}^2 \mid u(x, y) = C \} \) and \( \{ (x, y) \in \mathbb{R}^2 \mid v(x, y) = C \} \) for \( C = 0, \pm 1, \pm 2 \).

(b) Let \( (a, b) \neq (0,0) \) be a point in \( \mathbb{R}^2 \). The curves

\[
\begin{align*}
    u(x, y) &= x^2 - y^2 = a^2 - b^2, \\
    v(x, y) &= 2xy = 2ab,
\end{align*}
\]

clearly intersect at the point \( (a, b) \). Show that they intersect at right angles.

(c) If \( z = x + iy \), then \( z^3 = (x + iy)^3 = f(x, y) + ig(x, y) \) where \( f \) and \( g \) are polynomials of degree 3. Examine what happens where a level set of \( f \) and a level set of \( g \) intersect.

(d) Can you generalize the results from part (c)?

In addition, do the following problems from the text:

Section 8.14, \# 1, 2, 3, 4, 7, 10.
Section 8.17, \# 1, 2, 4, 5, 9, 11.
Section 8.22, \# 1, 2, 3, 4b, 5, 8, 9a, 12.