

Trigonometric Identities

1. General Identities:

- $\sin^2 \alpha + \cos^2 \alpha = 1$
- $1 + \tan^2 \alpha = \sec^2 \alpha$

2. Shifting and Reflecting Identities:

- $\sin(-\alpha) = -\sin(\alpha)$ (i.e. \sin is an odd function)
- $\cos(-\alpha) = \cos(\alpha)$ (i.e. \cos is an even function)
- $\sin(\alpha + k2\pi) = \sin(\alpha)$ and $\cos(\alpha + k2\pi) = \cos(\alpha)$ for all integer k .
- $\sin(\alpha + \pi) = -\sin(\alpha)$ and $\cos(\alpha + \pi) = -\cos(\alpha)$.
- $\sin(\alpha + \pi/2) = \cos(\alpha)$
- $\cos(\alpha + \pi/2) = -\sin(\alpha)$

3. Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

4. Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

5. Double and Half angle Formulas:

- $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) = 2\cos^2(\alpha) - 1 = 1 - 2\sin^2(\alpha)$
- $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$

•

$$\tan(2\alpha) = \frac{2\tan(\alpha)}{1 - \tan^2(\alpha)}$$

•

$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 + \cos(\alpha)}{2}}$$

•

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 - \cos(\alpha)}{2}}$$

•

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1 - \cos(\alpha)}{\sin(\alpha)} = \frac{\sin(\alpha)}{1 + \cos(\alpha)}$$

6. Angle Addition and Subtraction Formulas:

- $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
- $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
- $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
- $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

