Math 340: Review for Midterm2

(1) (25 pt) Let the matrix A be

$$A = \begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ -1 & 0 & 0 & 4 \end{bmatrix}.$$

(a) (10 pt) Solve equation $A\mathbf{x} = \mathbf{0}$.
(b) (10 pt) Find a basis for the solution space of the above equation.
(c) (10 pt) Find the nullity of A.

(2) (25 pt) Let P_2 be the vector space of all polynomials of degree ≤ 2.
(a) (5 pt) Write a basis for P_2.
(b) (10 pt) Given a set $S = \{t-1, t^2+1\}$ in P_2. Verify that S is linearly independent.
(c) (10 pt) Extend S to a basis for P_2.

(3) (20 pt) In the standard inner product space $\mathbb{R}^3$, let $S = \{[1,0,1]^T, [1,-2,2]^T\}$.
(a) (5 pt) Find the length of the vector $\vec{u}_1 = [1,0,1]^T$.
(b) (5 pt) Find the angle θ between the vectors $\vec{u}_1 = [1,0,1]^T$ and $\vec{u}_2 = [1,-2,2]^T$.
(c) (10 pt)Use the Gram-Schmidt process to obtain an orthonormal basis of the subspace $W = \text{Span}S = \text{Span}\{[1,0,1]^T, [1,-2,2]^T\}$.

(4) (20 pt) Let M_{22} be the real vector space of all 2×2 matrices.
(a) (10 pt) Is the subset W in M_{22} consisting of matrices $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ a subspace of M_{22}? Why? (Here a, b are real numbers.)
(b) (10 pt) If W is a subspace, find a basis for W. Then express $\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$ as a linear combination of the vectors in the basis.

(5) Consider the matrix

$$A = \begin{bmatrix} 5 & -3 & 1 \\ 1 & -2 & 1 \\ 3 & 1 & -1 \end{bmatrix}$$

(a) Find bases of the row space and column space of A respectively
(b) Find a basis of the null space of A
(c) Find the rank and the nullity of A
(d) Check if the vector $\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$ is in the column space of A.

(6) Consider the set W of all matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $a + d = 0$.
(a) Prove that W is a subspace of $M_{2\times 2}(\mathbf{R})$ (=the set of 2×2 matrices).
(b) Find a basis of W and its dimension of W.

(7) Check if the following set of polynomials is linearly independent

$$3 + x + x^2, \quad 2 + 2x + 5x^2, \quad 4 - 3x^2$$

(8) Let $S = \{\vec{v}_1, \vec{v}_2\}$ and $T = \{\vec{w}_1, \vec{w}_2\}$ be bases for a vector space V, and suppose that

$$\vec{v}_1 = -\vec{w}_1 + 4\vec{w}_2, \quad \vec{v}_2 = 5\vec{w}_1 - 3\vec{w}_2$$

(a) Find the change-of-coordinate matrix from S to T

(b) Find the coordinate vector $[\vec{x}]_S$ for $\vec{x} = 5\vec{v}_1 + 3\vec{v}_2$.

(9) Answer the following questions. You do not have to explain.

(a) Write down the zero vector in the vector space $\mathbb{P}_n$ of polynomials of degree less than equal to n.

(b) Let A be the matrix

$$A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$$

Its row echelon form is give by

$$R = \begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find bases of the column space and the row space respectively.

Glossary

You should know at least, if not all, the definitions of all these terms and understand what they stand for.

(a) Coefficient matrix, Augmented matrix

(b) Row echelon matrix, Reduced row echelon matrix, Elementary row operations

(c) Back substitutions, basic variables, free variables

(d) Vector spaces, zero vector, addition, scalar multiplication

(e) Subspace, definition of subspaces

(f) linear combination, linear independence, span,

(g) basis, dimension

(h) homogeneous equation

(i) coordinates, linear transformation, isomorphism, change of basis, transition matrix

(j) column space, row space, rank and nullity of a matrix

(k) inner product : definition, examples, Cauchy-Schwarz inequality, triangle inequality,

(l) orthogonal, orthonormal basis, Gram-Schmit process