Do ALL 9 problems and show ALL work.
Use only techniques that have been covered in class.

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<th>PROBLEM</th>
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1. (20 pts) Find the two volumes if the region in the $x,y$-plane bounded above by the parabola $y = 2 - x^2$, below $y = x^2$ and on the left by the $y$-axis is rotated about
   a) the $x$-axis,
   b) the $y$-axis.
2. (20 pts) Find the $y$-coordinate of the centroid of the curve given by the parametric equations

\[ x = \sqrt{3} t^2, \quad y = t - t^3, \quad 0 \leq t \leq 1 \]
3. (20 pts) Find the coordinates of the centroid of the finite region in the $x,y$-plane below the curve $y = 5x(1 - x^2)$ and above the $x$-axis.
4. a) (7 pts) Find the logarithmic derivative, $y'/y$, of the function

\[ y = \frac{\sqrt{1 + x^4} (\tan x)^2}{(1 + x^2)^3} \]

Do not simplify your answer.

b) (7 pts) Compute

\[ \int \frac{\sin x \cos x}{1 + \sin^2 x} \, dx \]

c) (6 pts) Compute

\[ \int \ln(x^{1/x}) \, dx \]
5. a) (10 pts) The population $y(t)$ of a certain bacterial colony satisfies the differential equation $dy/dt = (\ln 8) y$, with $y(0) = 100$. Find a formula for $y(t)$ and determine how long it takes for the population to double.

b) (10 pts) Compute

$$\lim_{x \to 0} (1 + 4x + x^2)^{1/(x^2)}$$
6. Compute
a) (5 pts) \[ \frac{d}{dx} x^3 e^{\sec x} \]

b) (5 pts) \[ \frac{d}{dx} x^\pi \]

c) (5 pts) \[ \int e^{2\ln x} \, dx \]

d) (5 pts) \[ \int xe^{x^2} \, dx \]
7. Compute
a) (6 pts) \[ \frac{d}{dx} \ln(1 + \cosh 3x) \]

b) (7 pts) \[ \int \frac{dx}{\sqrt{4 + 9x^2}} \]

c) (7 pts) \[ \int \frac{dx}{x^2 + 2x + 10} \]
8. (20 pts) Determine where the curve

\[ y = \frac{x - 1}{x^2} \]

is increasing, decreasing, concave up and concave down. Where are its local extrema and its inflection points. What about vertical and horizontal asymptotes. Use this information to sketch the curve on the coordinate system given below.
9. a) (5 pts) Find the average value of the function \( y = \frac{x^2 + 2}{x} \) in the interval [2, 4].

b) (5 pts) Find the average value of the function \( y = \frac{1}{\sqrt{1 - x^2}} \) in the interval [0, 1/2].

c) (10 pts) A particle moves in the \( x, y \)-plane with coordinates given by \( x = x(t) \) and \( y = y(t) \). If \( \theta \) denotes the angle between the positive \( x \)-axis and the line from the origin to the point \( (x, y) \), find a formula for \( d\theta/\ dt \) in terms of \( x, y, dx/dt, \) and \( dy/dt \). Evaluate this when \( x(t) = 1 + \cos 3t \) and \( y = \sin 3t \).