741 Home Work Problems - Set 1

RULES. You may not discuss these problems with anyone but me.
You can use your class notes, but no other reference material.
Home work due in class Tuesday Oct 6, unless you are or have been ill.

1. Show that a function $f\colon A \to B$ has a right inverse if and only if it is one-to-one, and it has a left inverse if and only if it is onto. Do any of these statements require the axiom of choice? Does the uniqueness of a one-sided inverse imply that f is invertible?

2. Let G be a not necessarily finite group and let H be a subgroup. Find a one-to-one correspondence between the left cosets of H and the right cosets. Conclude that the left index of H in G is the same as the right index. If K is another subgroup of G prove that $Hx \cap Ky$ is either empty or a right coset of $H \cap K$.

3. Let A and B be sets of the same size. In other words, there exists a one-to-one correspondence $\theta\colon A \to B$. Use this map to construct a group isomorphism from Sym_A to Sym_B.

4. These are four distinct problems. Here the latter problems build on the earlier ones. Show that $G = \mathrm{Sym}_n$ is generated by: (a) the $n-1$ transpositions $(1,2),(1,3),\ldots,(1,n)$; (b) the transposition $(1,2)$ and the $(n-1)$-cycle $(2,3,\ldots,n)$; (c) the $n-1$ transpositions $(1,2),(2,3),\ldots,(n-1,n)$; (d) the transposition $(1,2)$ and the n-cycle $(1,2,\ldots,n)$.

5. These are two distinct problems. Show that $G = \mathrm{Alt}_n$ is generated by: (a) all 3-cycles $(1,i,j)$ with $i < j$; (b) the $n-2$ three-cycles $(1,2,3),(1,2,4),(1,2,5),\ldots,(1,2,n)$.