

741 Home Work Problems - Set 2

RULES. You may not discuss these problems with anyone but me.

You can use your class notes, but no other reference material.

Home work due in class Tuesday Nov 24, unless you are or have been ill.

1. Find, up to isomorphism, all groups of order 12.
2. Let $G = \text{Sym}_m$ and let $n_p = n_p(G)$ denote the number of Sylow p -subgroups of G .
 - a. If $m = 4$, determine the isomorphism class of a Sylow 2-subgroup of G and compute n_2 and n_3 .
 - b. If $m = 5$, find n_2 , n_3 and n_5 .
3. Let G be a transitive permutation group on Ω and let $a \in \Omega$. Prove that G is 2-transitive if and only if $G = G_a \cup G_a g G_a$ for all $g \in G \setminus G_a$.
4. Let G be a group of order $280 = 2^3 \cdot 5 \cdot 7$ with a non-normal Sylow 7-subgroup P .
 - a. If $N = \mathbf{N}_G(P)$ is the normalizer of P in G , find $|N|$ and $n = |G : N|$.
 - b. Show that N is cyclic.
 - c. If n is the index obtained above, show that Sym_n does not contain a cyclic subgroup isomorphic to N .
 - d. Find the order of the kernel of the homomorphism $\theta: G \rightarrow \text{Sym}_n$ determined by the permutation representation of G on the right cosets of N .