Math 741 Midterm

RULES. Try to write efficiently and succinctly.
You can use your class notes, but no other reference material.
You may not discuss this exam with anyone but me.
Exams are due in class Tuesday October 25.

Let G be a finite, nonabelian simple group. Suppose H is a proper subgroup of G with index $|G : H| = n > 1$.

1. Show that G embeds isomorphically in Alt_n, and that $n \geq 5$.

2. If $n \leq 8$, prove that G has no elements of order 10.

3. Viewing G as a subgroup of Alt_n, show that either $G = \mathrm{Alt}_n$ or $|\mathrm{Alt}_n : G| \geq n$. Hence either $|G| = n!/2$ or $|G| \leq (n-1)!/2$.

4. Show that there is no nonabelian simple group of order 120. Indeed, if G is such a group, deduce from the above that $n \geq 8$, so $|H| \leq 15$. Now determine the number of Sylow 5-subgroups of G.

5. Show that the only nonabelian simple group of order 60 is Alt_5. To this end, suppose by way of contradiction that G is such a group and that G is not isomorphic to Alt_5. First, deduce from the above that $n \geq 6$, so $|H| \leq 10$. Next, show that G contains 24 elements of order 5. Finally, prove that a Sylow 2-subgroup P of G is a self-normalizing T.I. set, so G has 45 elements of order 2 or 4. (Recall that P is a T.I. set if distinct conjugates of P are disjoint. To prove that P is a T.I. set, you might consider the centralizer of the intersection of two such conjugates.)