

MATLAB 7

1. This session introduces you to *pplane6*, a useful piece of software for phase plane analysis. You can download it from our web site to add it as an m-file to your MATLAB software. Type:

`pplane6`

to enter. You will get the Setup Window which has a default system of equations built in together with an associated range of x and y values, a density of points for the direction field and some control over their display. Click on **Proceed** to get the Display Window. It pictures the direction field for the default system. If you click the mouse at a point, MATLAB uses numerical methods to calculate the solution curve through the point, first forward and then backward in time. Try this for a few sample points.

2. Return to the Setup Window and enter the system for the damped pendulum in the form

$$x' = y$$

$$y' = -\sin(x) - a * y$$

with initial choice of the damping parameter $a = 0$ (i.e. no friction). As display size take $-10, 10$ for x and $-4, 4$ for y together with 20 points in the direction field box. Go to the Display Window and click the mouse on $(0, 1)$. You should get a closed orbit. Next plot the orbits through $(0, 1.95)$, $(0, 2)$, and $(0, 2.05)$. It is difficult to do this using the mouse because the points are so close to each other but you can enter the initial points manually by first clicking on 'solutions' and then on 'Keyboard input'. (Before you click on 'Keyboard input', glance over the list of options for future reference). Now you can enter and compute the curves through these 3 points. Print the graph of these solutions. (As usual, use the `gtext` command to add your name, etc., to each printout.)

3. While you compute solutions as in (2), the program *pplane6* may make some comments on the bottom of the Display Window.

4. Return to the Setup Window and change a to 1, thereby adding friction to the system. Click on Proceed to modify the Display Window and replot the above 3 points to see how friction changes the response of the system.
5. Keeping $a = 1$, change the Setup Window to $(-20, 20)$ for x and $(-20, 20)$ for y . Click on Proceed and plot the effect of a large initial disturbance e.g. $(0, 15)$, as in an example in class. Print your output.
6. Pplane can also plot $x(t)$ and/or $y(t)$ as functions of t or give a 3D plot or even a composite of these. Use the Graph Menu on the pplane Display Window to experiment with this feature. Print the composite graph for the example in (5).
7. Return to the Setup Window, click on Gallery, and then on competing species. Modify the system given to get the following system with

$$\begin{aligned}
 0 &\leq x \leq 1, \quad 0 \leq y \leq 1 \\
 x' &= (1 - x - y)x \\
 y' &= (0.75 - 0.5x - y)y
 \end{aligned}$$

Click on Proceed. Examining the direction field, you can see roughly where there are equilibrium points. Click on Solutions and then on ‘Find an equilibrium point’. You are then instructed to click near a possible equilibrium point. If there are none nearby, you get a message indicating this; if there is one, a red dot appears around the point. In addition a new window labelled ‘PPlane Equilibrium point data’ appears. This window contains the following useful information:

- (a) A conservative classification of the equilibrium point. ‘Conservative’ here means that the underlying program keeps track of certain possible errors involved in the computations. If they are greater than a built-in tolerance, pplane will not classify the point, but if the tolerances are satisfied, pplane does classify the point.
- (b) The linear part of the system at the point, i.e. the matrix A for the system in coordinates about the point.

(c) The eigenvalues and eigenvectors of A .

Check out these other features at the four equilibrium points for this system beginning with $(0,0)$. Click on a few points in the Display Window to get a picture of the behavior of the system with IC in $0 \leq x \leq 1, 0 \leq y \leq 1$. Print your output.

8. Carry out the same process and get a printout for

$$x' = (1 - x - y)x$$

$$y' = (1.5 - x - y)y$$

for $0 \leq x \leq 1, 0 \leq y \leq 2$.

9. Hand in the graphs.