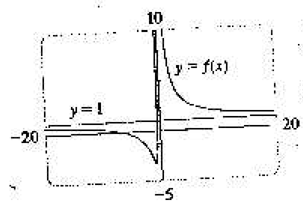
$$f(x) = \frac{x^2 + 7x + 3}{x^2}$$



EXAMPLE 4 Graph the function $f(x) = \sin(x + \sin 2x)$. For $0 \leq x \leq \pi$, locate all maximum and minimum values, intervals of increase and decrease, and inflection points correct to one decimal place.

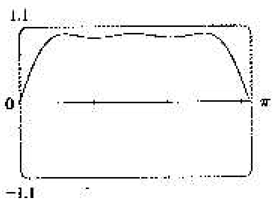


FIGURE 15

SOLUTION We first note that f is periodic with period 2π. Also, f is odd and $|f(x)| \leq 1$ for all x. So the choice of a viewing rectangle is not a problem for this function: we start with $[0, \pi]$ by $[-1.1, 1.1]$ (see Figure 15). It appears that there are three local maximum values and two local minimum values in that window. To confirm this and locate them more accurately, we calculate that

$$f'(x) = \cos(x + \sin 2x) \cdot (1 + 2\cos 2x)$$

and graph both f and f' in Figure 16. Using zoom-in and the First Derivative Test, we find the following values to one decimal place.

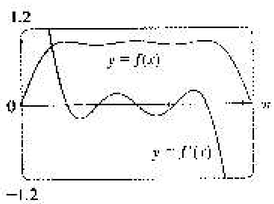


FIGURE 16

Intervals of increase:	$(0, 0.6), (1.0, 1.6), (2.1, 2.5)$
Intervals of decrease:	$(0.6, 1.0), (1.6, 2.1), (2.5, \pi)$
Local maximum values:	$f(0.6) \approx 1, f(1.6) \approx 1, f(2.5) \approx 1$
Local minimum values:	$f(1.0) \approx 0.94, f(2.1) \approx 0.94$

The second derivative is

$$f''(x) = -(1 + 2\cos 2x)^2 \sin(x - \sin 2x) - 4\sin 2x \cos(x + \sin 2x)$$

Graphing both f and f'' in Figure 17, we obtain the following approximate values:

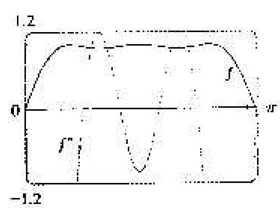
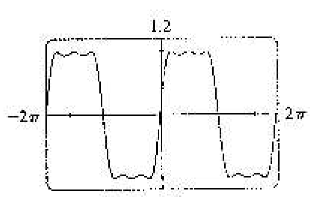
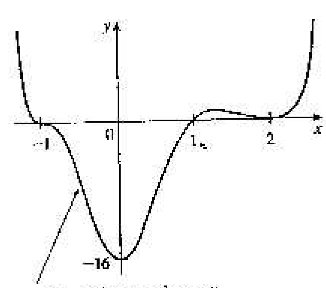


FIGURE 17

Concave upward on:	$(0.8, 1.3), (1.8, 2.3)$
Concave downward on:	$(0, 0.8), (1.3, 1.8), (2.3, \pi)$
Inflection points:	$(0, 0), (0.8, 0.97), (1.3, 0.97), (1.8, 0.97), (2.3, 0.97)$

Having checked that Figure 15 does indeed represent f accurately for $0 \leq x \leq \pi$, we can state that the extended graph in Figure 18 represents f accurately for $-2\pi \leq x \leq 2\pi$. ■





$y = (x-2)^4(x+1)^3(x-1)$