Mathematics Education Research Statement

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1. Background

Mathematics education is both a *science* and an *art*. Some learn the art of good teaching by example, characterizing education as a social science. Others validate their methods empirically via sound research. I do both, treating mathematics education as an empirical social science.

As an *art* and a social science, there is a saying "the best way to find out what makes a good teacher is to ask the good teachers" (cf. [1]). Many educators act as gardeners, giving student-plants free range while ensuring they get the nutrients they need to prosper. Others act as tomes of knowledge and culture which they pass to the next generation (cf. [1]). These opposite methods can produce similar results, which has led to the idea that "there are no proofs in mathematics education" (cf. [4]). The teacher's "special something" makes the difference.

In any *science*, empirical evidence is critical. While proofs are impossible without axioms, overwhelming evidence was sufficient to convince the world of Newton's Laws. Later Einstein disproved them, and presented enough evidence to convince scientists of his theory of relativity. Psychology was once linked to Freud's controversial artistic opinions but now has well-founded theories governing the limits of memory. Even mathematics was once an art, as is illustrated by the following passage from the popular math book Fermat's Enigma:

> "Pythagoras observed that Egyptians and Babylonians conducted each calculation in the form of a recipe that could be followed blindly. The recipes, which would have been passed down through the generations, always gave the correct answer and so nobody bothered to question them or explore the logic underlying the equations. What was important for these civilizations was that a calculation worked – why it worked was irrelevant" (cf. [5]).

The study of Mathematics Education as a science is its early stages. Skeptics cite lack of scope and replicability. Proponents cite studies revealing inadequacies in current methods. A middle-ground may be found. For credibility within mathematics circles, the researcher must first learn the mathematics before commenting on how to learn it. For credibility within education circles, a study must be done by someone competent in educational research. For change in either circle, one must follow a criteria such as in [4] to ensure that a study is credible and applicable. I am motivated to breach this divide between educators and mathematicians.

As a teaching assistant, I participated in numerous projects in mathematics education research. To understand learning communities I collaborated with a master's student in mathematics education and performed informal surveys and experiments. The first project which I designed and implemented employed techniques which facilitate interaction between teacher and students in a large lecture setting. I will address results of each project below.

2. Learning Communities

Learning Communities are usually formed informally as peer study groups. Studies have shown that minority students in the college environment (e.g. ethnically and culturally), who otherwise have the same motivation and aptitude, have high attrition rates. The Emerging Scholars Program, pioneered by Uri Treisman at UC Berkeley (now at UT Austin), is designed to provide a learning community for these individuals. The principles guiding everything from group sizes to student and teacher responsibilities have been and continue to be researched (cf. [2]).

The applicability of an educational study varies based on differences between the study group and implementing group. In the 2005-2006 academic year I taught a year long course of precalculus / calculus for emerging scholars. In Spring a student in mathematics education began work on a masters thesis in my course. Our collaboration focused on group dynamics and learning styles. Group composition, sizes, and roles were varied to maximize learning. A taxonomy of worksheet problem types was created to accommodate for student learning styles and confront student weaknesses. Many problems had “if you have time” parts to encourage fast students to delve *deeper* into challenging problems instead of completing *more* standard problems. Surveys, course evaluations, and grades revealed an improvement in student learning and fulfillment.

Motivation to think abstractly is difficult but important to foster. In the summer of 2006 I created a 30 minute motivation-inspiring presentation for the first day of an emerging scholars course. I focused on "mathematics as puzzle solving," relating the desire to solve Sudoku problems or create strategies in games to the desire of mathematicians to solve math problems. I was invited to give this presentation at a UW Madison Engineering and Chemistry TA training. In the Fall 2006 semester I went one step further in motivating students by asking emerging scholars to choose a subject of interest to them and give a presentation about it. Students gave short oral presentations to the class about Mathematics Education, Rubik's Cube, Game Theory, Programming, etc. Later they created poster boards about their projects and discussed them during a poster day with professors and graduate students. Students were inspired by this learning process. Since then, I have facilitated ongoing student projects to deepen motivation, and created a student-oriented webpage comparing Games, Puzzles, Magic, and Mathematics.

3. Large Lecture Techniques

Interaction between students and teachers is critical to learning. Student misunderstandings can often be settled by a simple question, but can be detrimental if allowed to fester. Learning communities, such as peer study homework groups, encourage students to resolve misunderstandings amongst each other. In large lecture settings, in-class group work (e.g. in the Emerging Scholars program) is infeasible. In Fall 2007, I designed and implemented an IRB[1]-approved controlled study of various large lecture techniques which facilitate student-teacher interaction.

In *Linear Algebra and Differential Equations* I designated two discussion sections as controls and two as experimental. A pretest guaranteed that the experimental sections were not predisposed to do better. The controls were taught in the usual manner of questions and answers, while the experimental sections were taught using three common techniques for increasing student-teacher interaction. Student-response systems allow teachers to quickly gauge class understanding. Random contact (also known as names-in-a-hat) replaces the dynamic of 10% of students asking 90% of questions by a dynamic of all students asking questions. Pre-class discussion questions replaces the dynamic of vocal students choosing discussion topics by a democratic nomination and vote before class starts. Assignment grades and survey results were compared between sections.

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1 IRB is the UW institutional review board for human subjects research.

This project will be presented at the Joint Mathematics Meetings of the AMS and MAA in 2008. While this project was of small scope, may not be applicable to other settings, and is not a blind study, it does establish a case study which a teacher may choose to try in a course setting. The scientific method is similar to what I would use in the future to test a new teaching technique. I look forward to future projects of greater scale assessing and comparing teaching strategies.

4. Future Research

In future studies I will make a few changes to the methods in the latter project. This project studied general students, while it would be more profitable to study different cultural groups. Teacher-specific bias may be countered by working in unison with other teachers and considering each course as one data point. A blind-study-administrator who selects experimental groups will eliminate study bias.

Learning Communities and techniques to maximize Student-Teacher Interaction have many avenues for future study. The wealth of other ideas warranting further study are endless. I am interested in initially studying high school and college courses and eventually broadening the scope of my research to K-8. Specific research topics will be governed both by personal motivation to improve the system and by current trends in education.

Research targeting specific cultural groups will allow teachers to identify their strengths, to build on, and weaknesses, to work with. At UW Madison, the main groups unfortunately segregate: ethnic minorities, east coasters (called "Coasties"), rural midwesterners, and city-dwelling midwesterners. Research studies should isolate each culture's differences to maximize applicability to different situations.

Psychological limitations and strengths are well studied and may be applied to education. Schoenfeld discusses (cf. [4]) the limitations on working memory which prevent even the best mathematicians from quickly multiplying 379 and 658 with their eyes closed. As an undergraduate I took a year-long course in Psychology and would have pursued a minor if a second was allowed. I am very interested working jointly with a psychology researcher to understand learning.

Good teachers deserve to be asked how they do it. Specific methods such as visual models and graphing programs can be easily isolated to assess student understanding. Accounts of a teacher's artistic talent may be published as a case-study alongside other case-studies with similar results, as in [3]. This allows for documentation of mathematics education as an art.

In the near future, I hope to improve my knowledge of mathematics education techniques. I have taken science education courses on Teaching-As-Research[2] and plan to pursue further mathematics-education-specific training. In any project I will use criteria for evaluating theories, like Schoenfeld's in [4], to maximize credibility. I look forward to collaborating with other educators to study cutting edge mathematics education techniques.

5. References

[1] Andrews, G. E. (2001). "*Review of Mathematics Education Research: A Guide for the Research Mathematician by Curtis McKnight, Andy Magid, Teri J. Murphy, Michelynn McKnight."* The American Mathematical Monthly, Vol. 108, No. 3 (Mar., 2001), pp. 281-285

[2] Asera, R. (2001). *Calculus and Community: A History of the Emerging Scholars Program.* College Board. http://www.collegeboard.com

[3] Friedberg, S. et al. (2001). *Teaching Mathematics in Colleges and Universities: Case Studies in Today's Classroom.* CBMS, Issues in Mathematics Education, Volume 10.

[4] Schoenfeld, A. H. (2000). *Purposes and Methods of Research in Mathematics Education.* Notices of the AMS. Vol. 41, No. 6 (Jun./July 2001), pp. 641-649.

[5] Singh, S. (1998). *Fermat's Enigma.* Anchor Books, 1st edition, 1998, ISBN: 0-385-49362-2.

2 Teaching-as-research involves the deliberate, systematic, and reflective use of research methods to develop and implement teaching practices that advance the learning experiences and learning outcomes of students/participants and teachers/facilitators. (cf. www.delta.wisc.edu)