1.8

Let Y be an affine algebraic variety of dim r in A^n.
Let H be a hyperplane in A^n, and assume $Y \not\subseteq H$.
Then Show every irred. comp. of $Y \cap H$ has dim $r-1$.

Let $Y \cap H = X_1 \cup \cdots \cup X_m$ irredundant irreducible decomp,
and $H = Z(f)$. (f an irred. poly in $k[x_1, \ldots, x_n]$).

Then for $\phi : A \xrightarrow{\text{canonical}} A(Y) = A/I(Y)$,
$\phi(f) \neq 0$ since then $f \in I(Y) = \ker \phi$,
$H = Z(f) \supseteq Y$, which
is contrary to our assumption.
Note $\phi(f)$ is still a polynomial, but in $A(Y)$.

Looking in $A(Y)$, $I(X_i)$ is a prime ideal, $(\forall i)$
and since $X_i \subseteq Y \cap H \subseteq H = Z(f)$,
we have $I(X_i) \supseteq (\phi(f))$

Take any ideal $\tilde{I} \supseteq (\phi(f))$, $\tilde{I} \subsetneq I(X_i)$.
Then $Y \cap H \supseteq Z(\tilde{I}) \supsetneq X_i$.
Hence $\bigcup_{i=1}^{m} (Z(\tilde{I}) \cap X_i)$ (disjoint union) is an irred. decomp. of $Z(\tilde{I})$.
This decomp. is nontrivial since $Z(\tilde{I}) \cap X_i = X_i \subsetneq Z(\tilde{I})$
Hence $Z(\tilde{I})$ is not irred, i.e. $\tilde{I}$ is not prime.

Hence $I(X_i)$ is the minimal prime above $(\phi(f))$,
so by Krull's Hauptidealsatz (Thm 1.11A), $ht\, I(X_i) = 1$.

Hence $\dim X_i = \dim A(Y) / \phi(I(X_i)) = \dim A(Y) - ht(\phi(X_i))$
$= r - 1$.

$\square$.