We have

$$\begin{tikzcd} t \arrow[r, mapsto] & (t, t^2, t^3) \\ v: \mathbb{A}^1 \arrow[r] \arrow[d, phantom, "\cap"] & Y \subseteq \mathbb{A}^3 \arrow[d, phantom, "\cap"] \\ \tilde{v}: \mathbb{P}^1 \arrow[r] & \overline{Y} \subseteq \mathbb{P}^3 \\ (x_0 : x_1) \arrow[r, mapsto] & (x_0^3 : x_0^2 x_1 : x_0 x_1^2 : x_1^3). \end{tikzcd}$$

$$Y_0 \quad Y_1 \quad Y_2 \quad Y_3 \leftarrow \text{new labels.}$$

Let $X = Z(\langle Y_1 Y_2 - Y_0 Y_3,\ Y_1^2 - Y_2 Y_0,\ Y_2^2 - Y_1 Y_3 \rangle)$
$= Z(\langle \beta(xy - z),\ \beta(x^2 - y),\ \beta(xz - y^2) \rangle)$
(where $Y_1 = x$, $Y_2 = y$, $Y_3 = z$).

Clearly $X \supseteq \overline{Y}$.

But take any $(a:b:c:d) \in X$

Then either $a \neq 0$ or $d \neq 0$

(Since if $a = d = 0$, then by $\underset{b}{Y_1}^2 - \underset{c}{Y_2}\,\underset{a}{Y_0} = 0$ we get $b = 0$,
and by $\underset{c}{Y_2}^2 - \underset{b}{Y_1}\,\underset{d}{Y_3} = 0$ we get $c = 0$, so
$(a:b:c:d) = (0:0:0:0) \notin \mathbb{P}^3$).

case i: $a = 0$, $d \neq 0$.

Then $\underset{b}{Y_1}^2 - \underset{a}{Y_0}\,\underset{c}{Y_2} = 0 \Rightarrow b = 0$

and so $\underset{c}{Y_2}^2 - \underset{b}{Y_1}\,\underset{d}{Y_3} = 0 \Rightarrow c = 0$,

so that $a, b, c = 0$.

projective space, ratio d^2

Then $(a:b:c:d) = (0:0:0:d) = (0:0:0:d^3)$
$= \tilde{v}(0:0:0:d)$

$\Rightarrow (a:b:c:d) \in \overline{Y}$.

case ii: $a \neq 0$.

Then $v(a:b) = (a^3 : a^2 b : a b^2 : b^3)$.

But $\underset{b}{Y_1}^2 - \underset{c}{Y_2}\,\underset{a}{Y_0} = 0 \Rightarrow ab^2 = ca^2$, and $b^3 = bca$

and ~~$Y_2^2 - Y_1 Y_3 = 0$~~ $\underset{b}{Y_1}\,\underset{c}{Y_2} - \underset{a}{Y_0}\,\underset{d}{Y_3} = 0 \Rightarrow (bc)a = (ad)a$, so $v(a:b) = (a^3 : a^2 b : a^2 c : a^2 d)$
$= (a:b:c:d) \Rightarrow (a:b:c:d) \in \overline{Y}$. Δ.