

Math 221, Quiz 6, 17 Nov 2000

Answers

1 If f and g are two continuous functions on $[a, b]$ and if for all x in $[a, b]$ $f(x) \geq g(x)$, then the area of the region between f and g over $[a, b]$ is

- (a) $\int_b^a [g(x) - f(x)] dx$ (b) $\int_a^b [g(x) - f(x)] dx$ (c) $\int_a^b f(x)g(x) dx$ (d) $2 \int_0^b [g(x) - f(x)] dx$

Answer: The area of the region is $\int_a^b [f(x) - g(x)] dx = \int_b^a [g(x) - f(x)] dx$. So (a) is the correct answer.

2 Gilfong's favorite region in the whole wide world is over $[0, \frac{\pi}{4}]$ bounded above by $\cos x$ and below by $\sin x$. He would like to know the area of this region so that he can shout it from rooftops. What should Gilfong shout?

- (a) $\sqrt{2} - 1$ (b) $1 - \sqrt{2}$ (c) $1 + \sqrt{2}$ (d) $\sqrt{2}$

Answer: The area of the region is

$$\begin{aligned} \int_0^{\frac{\pi}{4}} [\cos x - \sin x] dx &= \sin x + \cos x \Big|_0^{\frac{\pi}{4}} \\ &= \sin \frac{\pi}{4} + \cos \frac{\pi}{4} - (\sin 0 + \cos 0) \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0 + 1) \\ &= \sqrt{2} - 1. \end{aligned}$$

So (a) is the correct answer.

3 Rikitikitambonosarambocheriberibuchipitperipambo skateboards around town with velocity $v(t) = 69(t - 1) \frac{\text{cm}}{\text{sec}}$ and boasts, "I go real far real fast." How far does Riki really go between $t = 0$ and $t = 2$, that is, what's his total distance traveled in that interval?

- (a) 69 cm (b) $\frac{1}{2} \cdot 69$ cm (c) 0 cm (d) $69 \frac{\text{cm}}{\text{sec}^2}$

Answer: The total distance traveled is

$$\begin{aligned}\int_0^2 |v(t)| dt &= \int_0^2 |69(t-1)| dt \\ &= 69 \int_0^2 |t-1| dt \\ &= 69\left(\frac{1}{2} + \frac{1}{2}\right) \quad (\text{by graphing } |t-1| \text{ or just computing}) \\ &= 69.\end{aligned}$$

So (a) is the correct answer.

4 Hmm, I seem to be in a muddle. Could you set me straight, and tell me the average value of the function $f(x) = a - b$ on $[a, b]$?

(a) $a - b$ (b) $b - a$ (c) c (d) $-a - b$

Answer: By computing $\frac{1}{b-a} \int_a^b f(x) dx$ or remembering that the average value of a constant function on an interval is that constant, you get an average value of $a - b$. So (a) is the correct answer.

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There are 169 scores

score	count	percent
20	18	10.7%
15	46	27.2%
10	52	30.8%
5	39	23.1%
0	14	8.3%

Mean score = 10.4.

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