Math 221, Quiz VII, December 1, 2000
Answers

The area bounded by the curve $y = x^2$ and the line $y = 4$ generates various solids of revolution as indicated below. For each one write a definite integral for the volume generated. DO NOT EVALUATE THE INTEGRAL BUT BE SURE TO SPECIFY THE LIMITS OF INTEGRATION. (This is essentially Problem 18 in Section 5-5.)

1) about the y-axis (i.e. the line $x = 0$)

Answer: By disks:

$$V = \int_{y=0}^{4} \pi(\sqrt{y})^2 \, dy.$$

By shells:

$$V = \int_{x=0}^{2} 2\pi x(4 - x^2) \, dx.$$

2) about the line $x = 2$.

Answer: By washers:

$$V = \int_{y=0}^{4} \pi\big((2 + \sqrt{y})^2 - (2 - \sqrt{y})^2\big) \, dy.$$

By shells:

$$V = \int_{x=-2}^{2} 2\pi(2 - x)(4 - x^2) \, dx.$$

3) about the x axis (i.e. the line $y = 0$).

Answer: By washers:

$$V = \int_{x=-2}^{2} \pi\big(4^2 - x^4\big) \, dx$$

By shells:

$$V = \int_{y=0}^{4} 2\pi y\big(2\sqrt{y}\big) \, dy.$$

4) about the line $y = 4$.

Answer: By disks:

$$V = \int_{x=-2}^{2} \pi\big((4 - x^2)^2\big)\, dx$$

By shells:

$$V = \int_{y=0}^{4} 2\pi(4 - y)\big(2\sqrt{y}\big)\, dy.$$

5) about the line $y = -1$.

Answer: By washers:

$$V = \int_{x=-2}^{2} \pi\big(5^2 - (1 + x^2)^2\big)\, dx$$

By shells:

$$V = \int_{y=0}^{4} 2\pi(y + 1)\big(2\sqrt{y})\big)\, dy.$$

Grader's Comments. The performance was rather weak. Apparently many students thought that the region being rotated changed when the axis of rotation changed. Encourage the students to label points on the graph with both coordinates - e.g. (x, x^2) or $(\sqrt{y}, y)$

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There are  173 scores
      range        count       percent
     18... 20      23          13.3%
     16... 17       8           4.6%
     14... 15       4           2.3%
     12... 13      15           8.7%
     10... 11      12           6.9%
      8...  9      19          11.0%
      0...  7      92          53.2%
Mean score =   7.5.
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