Let $f(x) = \frac{x^3 + 1}{x}$.

a) (10 points) Find $f'(x)$, the derivative of $f$ at $x$.

**Answer:** We can use the quotient rule, along with the addition and power rules:

$$f'(x) = \frac{(x^3 + 1)'x - (x^3 + 1)(x)'}{x^2} = \frac{(3x^2 + 0)x - (x^3 + 1)(1)}{x^2}$$

$$= \frac{3x^3 - (x^3 + 1)}{x^2} = \frac{2x^3 - 1}{x^2},$$

or we can do some algebra first:

$$f(x) = \frac{x^3 + 1}{x} = \frac{x^3}{x} + \frac{1}{x} = x^2 + x^{-1}$$

and then take the derivative using the addition and power rules:

$$f'(x) = (x^2 + x^{-1})' = (x^2)' + (x^{-1})' = 2x - x^{-2} = 2x - \frac{1}{x^2}.$$  

b) (10 points) Find all the points $P(a, f(a))$ where the tangent line to the curve $y = f(x)$ is horizontal, and write down the equation of the tangent line at such points.

**Answer:** The tangent line to the curve $y = f(x)$ at the point $(a, f(a))$ has slope $f'(a)$. Since horizontal lines are precisely the lines with slope 0, we are interested in the points of the form $(a, f(a))$ with $a \neq 0$ and $f'(a) = 0$.

$$f'(a) = 0 \iff \frac{2a^3 - 1}{a^2} = 0 \iff 2a^3 - 1 = 0 \iff a = \frac{1}{\sqrt[3]{2}}.$$  

As $f\left(\frac{1}{\sqrt[3]{2}}\right) = \frac{3\sqrt[3]{2}}{2}$, it follows that the only point where the tangent line to the curve $y = f(x)$ is horizontal is $\left(\frac{1}{\sqrt[3]{2}}, \frac{3\sqrt[3]{2}}{2}\right)$, and the equation of the tangent line at that point is

$$y = \frac{3\sqrt[3]{2}}{2}.$$