(I) Define the following terms: supremum (least upper bound), infimum (greatest lower bound), accumulation point, open set, closed set, compact set, continuous function, uniformly continuous function, Cauchy sequence, convergent sequence.

(II) Simplify the following expressions:

(a) \( \bigcap_{n=1}^{\infty} \left( -\frac{1}{n}, \frac{1}{n} \right) \)  
(b) \( \bigcup_{n=1}^{\infty} \left( -\frac{1}{n}, \frac{1}{n} \right) \)  
(c) \( \bigcap_{n=1}^{\infty} (-n,n) \)

(d) \( \bigcup_{n=1}^{\infty} (-n,n) \)  
(e) \( \bigcup_{n=1}^{\infty} \left( -\frac{1}{n}, \frac{1}{n} + \frac{1}{n} \right) \)  
(f) \( \bigcap_{n=1}^{\infty} \left( -\frac{1}{n}, \frac{1}{n} + \frac{1}{n} \right) \)

(g) \( \mathbb{R} \setminus \bigcup_{n=1}^{\infty} (-n,n) \)  
(h) \( \mathbb{R} \setminus \bigcup_{n=1}^{\infty} \left( -\frac{1}{n}, \frac{1}{n} + \frac{1}{n} \right) \)  
(i) \( \mathbb{R} \setminus \bigcap_{n=1}^{\infty} \left( -\frac{1}{n}, \frac{1}{n} + \frac{1}{n} \right) \)

(III) Assume that the set \( X \subset [a, b] \) has the following properties:

(1) \( a \in X \);

(2) for all \( x \in [a, b] \) we have \([a,x] \subset X \implies [a,x] \subset X \);

(3) for all \( x \in [a, b] \) we have \([a,x] \subset X \implies \exists y > x \text{ such that } [a,y] \subset X \).

Show that \( X = [a, b] \).

(IV) Assume that \( \{x_n\}_{n=1}^{\infty} \) is a bounded above (i.e. there is an \( M \in \mathbb{R} \) with \( x_n \leq M \) for all \( n \)) and increasing (i.e. \( x_n \leq x_{n+1} \) for all \( n \)) sequence. Prove that \( \{x_n\}_{n=1}^{\infty} \) converges.

(V) State and prove the Bolzano Weierstrass Theorem.

(VI) State and prove the Heine Borel Theorem.

(VII) Prove that a continuous function on a closed interval is uniformly continuous.

(VIII) Prove the Max Min Theorem: If \( f : [a, b] \to \mathbb{R} \) is continuous, there exists \( p, q \in [a, b] \) with \( f(p) \leq f(x) \leq f(q) \) for all \( x \in [a, b] \).

(IX) Prove the Intermediate Value Theorem: If \( f : [a, b] \to \mathbb{R} \) is continuous and \( f(a) < y_0 < f(b) \) then there exists \( x \in [a, b] \) such that \( y = f(x) \).

(X) Suppose that \( f : [a, b] \to \mathbb{R} \) is continuous and that \( c \in [a, b] \) is such that \( f(a) < f(b) < f(c) \). Show that \( f \) is not one-one.

(XI) Let \( \bar{D} \) denote the closure of the set \( D \subset \mathbb{R} \), i.e. \( x \in \bar{D} \) iff there exists a sequence \( \{x_n\}_{n=1}^{\infty} \) converging to \( x \) with \( x_n \in D \) for all \( n \). Suppose that \( f : D \to \mathbb{R} \) is uniformly continuous. Show that there exists a unique continuous function \( F : \bar{D} \to \mathbb{R} \) such that \( f(x) = F(x) \) for \( x \in D \). Give an example which shows this is false if \( f \) is only assumed to be continuous.