1. Find a 1-1 function from the set \( J := \{1, 2, 3, 4, \ldots\} \) of natural numbers onto the set \( S := \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\} \) of odd integers.

**Answer.** There are many possible answers. Perhaps the easiest is to put the even natural numbers in 1-1 correspondence with the positive odd integers and the odd natural numbers in 1-1 correspondence with the negative odd integers:

\[
f(n) := \begin{cases} 
    n - 1 & \text{if } n \in J \text{ is even}, \\
    -n & \text{if } n \in J \text{ is odd}. 
\end{cases}
\]

Note that this same function can be defined by a single formula as

\[
f(n) = \left( \frac{1 + (-1)^n}{2} \right)(n - 1) - \left( \frac{1 - (-1)^n}{2} \right)n
\]

but the first definition is easier to understand.

2. Often, if the domain is not specified, it is assumed to be the set of all real numbers for which the formula defining the function defines a real number. For the function defined by

\[
f(x) = \frac{x}{x + 2}
\]

specify its domain and its image \( \text{im}(f) \). Is \( f \) injective (i.e. 1-1)? If so, find the inverse function \( f^{-1} \) and specify its domain and image. If not, specify points \( x_1 \) and \( x_2 \) with \( x_1 \neq x_2 \) but \( f(x_1) = f(x_2) \).

**Answer.** If \( y = f(x) \) then

\[
y = \frac{x}{x + 2}
\]

so \( y(x + 2) = x \) so \( x(y - 1) = -2y \) so

\[
x = \frac{2y}{1 - y}
\]

This proves that \( f \) is 1-1 and computes the inverse

\[
f^{-1}(y) = \frac{2y}{1 - y}
\]

at the same time. By Theorem 0.6 in the text we have

\[
\text{im } f = \text{dom } f^{-1} = \{ y \in \mathbb{R} : y \neq 1 \}
\]

and

\[
\text{im } f^{-1} = \text{dom } f = \{ x \in \mathbb{R} : x \neq -2 \}.
\]