Syllabus for Math 421 (Third Draft)

Prof. J. Robbin

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When and Where: 11:00 MWF, B105 Van Vleck

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My Office Hours: 9:30-11 T, 9-10 W, 11-12:30 R. I will also see students at other times on TR morning if I am in my office. If want to seem me but can’t come at these times, send me an email to set up an appointment.


Overview

This course was instituted as a “bridge course” between the problem oriented calculus courses (221,222,234) and the proof oriented advanced course (521). As I see it, this course is difficult because:

- Students must learn a new language where difficult words like if, and, or, not, for all, there exists must be used carefully.
- Students must learn a new notation: set theoretic notation.
- Students must learn to use inequalities carefully.
- Students must learn to read carefully.
Students must learn to write proofs.

A proof is an argument intended to convince the reader that a general principle is true in all situations. The amount of detail that an author supplies in a proof should depend on the audience. Too little detail leaves the reader in doubt; too much detail may leave the reader unable to see the forest for the trees. As a general principle, the author of a proof should be able to supply the reader with additional detail on demand. When a student writes a proof for a teacher, the aim is usually not to convince the teacher of the truth of some general principle (the teacher already knows that), but to convince the teacher that the student understands the proof and can write it clearly.

A correct proof is a sequence of steps each of which is either a special case of a previously established general principle or results from earlier steps in the proof by applying a logical rule of inference. The previously established principles may be axioms, which means that they are assumed without proof. The virtue of mathematical reasoning is that (in principle at least) it makes explicit these axioms and that its conclusions are correct in any situation where its hypotheses are correct. What distinguishes mathematical reasoning from other forms of intellectual inquiry is the precision of mathematical statements. In courses like mathematical logic (e.g. philosophy 211 and math 571) “formal languages” are developed to explain correct mathematical reasoning and to make it possible to study mathematical reasoning using the methods of mathematical reasoning, but these formal languages are too cumbersome to be used for communicating real mathematics.

Policy

The purpose of the course is to teach students to read and write proofs. Your grade will be based in large part upon the clarity of your written work. You will have to learn most of the material on your own by reading the text book. It is more important that you learn a little bit well than a lot poorly.

I hope to go through the book at a rate of one chapter every two weeks. Each Wednesday you will hand in fifteen problems of your choosing from the problems at the end of the chapter being studied. Doing a “project” counts for seven problems. You may redo a problem for which you were not given credit in the previous week. You should do all of the starred exercises, since they will be used subsequently in the text.
Each Friday you will have a quiz. The lowest two quizzes will be dropped and there will be no makeups. There will be an hour exam in class on the Friday of the seventh week. The final is 2:45 Thursday, May 13 (exam period 7). It is guaranteed that 80% of the exam questions and all of the quiz questions will be chosen from the book, either problems from the end of the chapter or statements and/or proofs of theorems from the text. The homework, in-class work (i.e. quizzes and hour exam), and final exam each represent about one third of the final grade.

Each piece of written work you do in this class should be self contained. This means that aside from standard notations (like \( \mathbb{R} \) for the set of real numbers) you should explain your notation. (However, you shouldn’t repeat a definition that appears in the statement of a quiz question in your answer to that question.) In homework, you should not assume that the reader has the book, so you should not simply write an answer with no indication of what the problem is. You may however refer to the text as in the phrase “By Theorem 3.2 of the text…” When you write, imagine that you are writing not to convince a grader that you understand but rather to convince a potential reader at your level of the validity of your arguments. Homework should be legible, stapled together, and in final draft form. This last means that there should not be too many corrections on the page. Show some respect for the reader!

The grader is Boian Popunkiov. His mail box (on the second floor of Van Vleck) is #47. He will try to grade problems on the weekend so they can be returned on the Monday following the Wednesday on which they are turned in. He may grade before the weekend, but said he would accept problems turned in to him before he does the grading.