Every Triangle is Isosceles!?

Let \( ABC \) be a triangle; we will prove that \( AB = AC \). Let \( O \) be the point where the perpendicular bisector of \( BC \) and the angle bisector at \( A \) intersect, \( D \) be the midpoint of \( BC \), and \( R \) and \( Q \) be the feet of the perpendiculars from \( O \) to \( AB \) and \( AC \) respectively (see figure).

The right triangles \( ODB \) and \( ODC \) are congruent since \( OD = OD \) and \( DB = DC \). Hence \( OB = OC \). Also the right triangles \( AOR \) and \( AOQ \) are congruent since \( \angle RAO = \angle QAO \) (\( AO \) is the angle bisector) and \( \angle AOR = \angle AOQ \) (the angles of a triangle sum to 180 degrees) and \( AO \) is a common side. Hence \( OR = OQ \). The right triangles \( BOR \) and \( COQ \) are congruent since we have proved \( OB = OC \) and \( OR = OQ \). Hence \( RB = QC \). Now \( AR = AQ \) (as \( AOR \) and \( AOQ \) are congruent) and \( RB = QC \) (as \( BOR \) and \( COQ \) are congruent) so \( AB = AR + RB = AQ + QC = AC \) as claimed.