

# A model for the collapse of the twin towers

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This note represents our attempt to reproduce the reasoning behind the “pancake theory” of the collapse of the twin towers on 9/11. Our starting point is the following quote from [5] (click).

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*The height of the South Tower is 1362 feet. I calculated that from that height, freefall in a vacuum (read, absolutely no resistance on earth) is 9.2 seconds. According to testimony provided to the 9-11 Commission, the tower fell in 10 seconds. Other data shows it took closer to 14 seconds. So the towers fell within 0.8-4.8 seconds of freefall in a vacuum. Just like WTC7, this speed seemed impossible if each of the 110 floors had to fail individually.*

*As I was considering this, another problem arose. There is a principle in physics called the Law of Conservation of Energy. There is also the Law of Conservation of Momentum. I'll briefly explain how these principles work. Let's assume there are two identical Honda Civics on the freeway. One is sitting in neutral at a standstill (0 mph). The other is coasting at 60 mph. The second Honda slams into the back of the first one. The first Honda will then instantaneously be going much faster than it was, and the second will instantaneously be going much slower than it was.*

*This is how the principle works in the horizontal direction, and it works the same in the vertical direction, with the added constant force of gravity added to it. Jim Hoffman, a professional scientist published in several peer-reviewed scientific journals, took a long look at all of this. He calculated that even if the structure itself offered no resistance, that is to say, even if the 110 floors of each tower were hovering in mid-air, the “pancake” theory would*

*still have taken a minimum of 15.5 seconds to reach the ground. So, even if the building essentially didn't exist, if it provided no resistance at all to the collapse, just the floors hitting each other and causing each other to decelerate would've taken 15.5 seconds to reach the ground.*

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A body in free fall with an initial velocity of  $v$  falls a distance of

$$d = vt + \frac{gt^2}{2} \tag{1}$$

in time  $t$ . By the quadratic formula,

$$t = \frac{-v + \sqrt{v^2 + 2gd}}{g} \tag{2}$$

We interpret the above “Honda collision law” to mean that when a body of mass  $M$  with velocity  $w$  hits a body of mass  $m$  at rest the two bodies become a single body of mass  $(M + m)$  with velocity  $v$  determined by

$$Mw = (M + m)v. \tag{3}$$

(This is called a *completely inelastic collision* on page 27 of [4].) We assume that there are  $N = 110$  floors and that the  $k$ th floor has a mass of  $m_k$ . (Later we will assume that all the  $m_k$  are the same.) We will assume that the collapse began at the  $L$ th floor and we will do our calculation for various values of  $L$ .<sup>1</sup> Let  $d_k$  be the distance from the  $k$ th floor (the ground floor is the zeroth floor) to the one below; later we will take  $d_k = 1362/N$ . Let  $v_k$  denote the initial velocity when the  $k$ th floor begins to collapse,  $t_k$  the time from that moment until it hits the floor below, and  $w_k$  be its velocity just as it hits the floor below. Then by (1)

$$d_k = v_k t_k + \frac{gt_k^2}{2} \tag{4}$$

and differentiating gives

$$w_k = v_k + gt_k. \tag{5}$$

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<sup>1</sup> According to [3](click) “... the jet that hit the South Tower, 18 minutes later, veered to the right just before impact and hit the rightmost third of the tower’s southwest wall, producing an impact gash that extended from the 78th to the 94th floor.”

After collision with the next floor Equation (3) gives the next initial velocity

$$(m_k + \cdots + m_N)w_k = (m_{k-1} + m_k + \cdots + m_N)v_{k-1}. \quad (6)$$

From (2) we have

$$t_k = \frac{-v_k + \sqrt{v_k^2 + 2gd_k}}{g} \quad (7)$$

and Equations (5) and (6) give

$$v_{k-1} = \frac{m_k + \cdots + m_N}{m_{k-1} + m_k + \cdots + m_N}(v_k + gt_k) \quad (8)$$

Now assume that all the  $d_k$  are the same, namely

$$d_k = d := \frac{1362}{110} \quad (9)$$

and that all the  $m_k$  are the same, say  $m_k = m$ . Since  $m$  cancels we take  $m_k = m = 1$ . Thus formulas (7) and (8) simplify to

$$t_k = \frac{-v_k + \sqrt{v_k^2 + 2gd}}{g} \quad (10)$$

and

$$v_{k-1} = \frac{N-k}{N-k+1}(v_k + gt_k) \quad (11)$$

with  $v_L = 0$ . At the point when the  $L$ th floor hits the the first floor it has a velocity of  $v_1$  and the top of the building is still a distance of  $(N-L+1)d$  above the ground so the additional time until the top of the building hits the ground is by (2)

$$\tilde{T} = \frac{-v_1 + \sqrt{v_1^2 + 2g(N-L+1)d}}{g}. \quad (12)$$

The total time to collapse is  $t_1 + \cdots + t_{L-1} + \tilde{T}$ .

Here is the program to compute the time to collapse.

```
#!/usr/bin/env python
from math import *
g=32.0
N=110
d=1362.0/N
for L in [1,78,94,110]:
    k=L; v=0; T=0
    while k>1:
        t=(-v+sqrt(v*v+2*g*d))/g
        T=T+t
        v=((N-k)*(v+g*t))/(N-k+1)
        k=k-1
    T=T+(-v+sqrt(v*v+2*g*(N-L+1)*d))/g
    print "L=",L," T=",T
```

The output is

```
L= 1    T= 9.22632104362
L= 78   T= 11.4905116965
L= 94   T= 12.5051575088
L= 110  T= 15.8381584163
```

The time required to free fall from a height of 1362 is

$$\sqrt{2 * 1362/g} = 9.22632104362$$

The above ignores the fact that the top of the building actually came to rest on a pile of rubble five stories high. We can estimate this by stopping the loop at  $k = 6$  and replacing  $(N-L+1)*d$  by  $(N-L+1-5)*d$  in the penultimate line. The output is

```
L= 1    T= 9.01419335372
L= 78   T= 10.9466312672
L= 94   T= 11.9126376955
L= 110  T= 15.1611285286
```

The websites [1] (click) and [2] (click) contain explanations by professors of civil engineering written shortly after the attack. The website [6] (click), written by a physicist, criticizes these two articles.

After this article was written we found a similar program [7] (click) on the web. The underlying assumptions are the same as ours. Here is a quote from the ambient website.

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*This program computes total collapse times based on parameters describing the floor on which the collapse started. The program makes the following assumptions, all of which favor short collapse times.*

- *Each floor is an infinitely thin slab, and all the mass of a story is concentrated in the slab.*
- *Mass is uniformly distributed among the stories.*
- *The overhanging portion (eg: 14 floors in the North Tower) falls as a block, with its bottom floor accumulating pancaked slabs of the once-intact floors as it encounters them.*
- *Once the bottom of the block reaches the ground, the floors in it start to pancake from bottom to top, the roof of the tower falling at freefall at that point.*
- *The falling block remains perfectly centered over the intact portion.*
- *The accumulation of floors is inelastic.*
- *Each floor's support vanishes when touched by the falling block.*
- *Momentum is conserved.*
- *None of the kinetic energy of the falling mass is diverted to other sinks (concrete pulverization, steel bending, etc.)*

*The following table summarizes the results of running the program with parameters specifying that the collapse starts at the 80th and 95th floors:*

<i>start floor</i>	<i>crash zone to ground</i>	<i>roof to ground</i>
	<i>(seconds)</i>	<i>(seconds)</i>
80	9.733	11.613
95	11.604	12.608

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## References

- [1] Thomas W. Eager & Christopher Musso: Why Did the World Trade Center Collapse? Science, Engineering, and Speculation, *Journal of the Minerals, Metals and Materials Society*, 53/12:8-11 (2001). [www.tms.org/pubs/journals/JOM/0112/Eagar/Eagar-0112.html](http://www.tms.org/pubs/journals/JOM/0112/Eagar/Eagar-0112.html)
- [2] Zdenek P. Bazant & Yong Zhou: Why Did the World Trade Center Collapse? Simple Analysis, *J. Eng. Mech.* 129, 839 (2003) [www.tam.uiuc.edu/news/200109wtc/](http://www.tam.uiuc.edu/news/200109wtc/) Addendum *J. Eng. Mech.* 128:369, March 2002.
- [3] [www.911review.com/attack/wtc/impacts.html](http://www.911review.com/attack/wtc/impacts.html)
- [4] Arnold Sommerfeld: *Lectures in Theoretical Physics, I: Mechanics*, (Translated from a German text written in 1942), Academic Press, 1964.
- [5] Dave Heller: Taking a Closer Look: Hard Science and the Collapse of the World Trade Center. This article originally appeared in *Garlic and Grass* and may be found in the Physics 911 website at [www.physics911.net/closerlook.htm](http://www.physics911.net/closerlook.htm)
- [6] Steven E. Jones: Why Indeed Did the WTC Buildings Collapse? [www.physics.byu.edu/research/energy/htm7.html](http://www.physics.byu.edu/research/energy/htm7.html)
- [7] 9 - 11 Research, [www.911research.wtc7.net/wtc/models](http://www.911research.wtc7.net/wtc/models). The program is at [www.911research.wtc7.net/wtc/models/docs/Collapse.java](http://www.911research.wtc7.net/wtc/models/docs/Collapse.java)