Residual distribution schemes for Bondi-Hoyle accretion

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Black hole accretion is a key component in astrophysical phenomena such as gamma-ray bursts and quasars. As a step towards understanding such phenomena, accurate and efficient computational methods for simulating gas dynamics in the presence of black holes must be developed. In this work a residual distribution scheme on unstructured grids is constructed for solving the general relativistic version of the compressible Euler equations. In particular, we simulate Bondi-Hoyle accretion onto a Schwarzschild black hole. Presented here are preliminary results in a larger effort to develop high-order methods on unstructured meshes for general relativistic flows.

1 Equations of general relativistic hydrodynamics

The Einstein equations of general relativity relate how matter interacts with the geometry of spacetime. In many astrophysical applications, such as in the accretion of matter onto black holes, the basic spacetime geometry induced by the compact object of interest is not significantly altered by the dilute gas with which it is interacting. In such a scenario, the full Einstein equations are often simplified to the so-called *test-fluid* equations in which the background spacetime is frozen and only the matter equations are evolved.

We follow the treatment of Martí et al. [5] (also see Font [3] for a review) in that we decompose the background metric into its 3+1 ADM [2] form:

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = \left(\beta^i \beta_i - \alpha^2\right)\, dx^0\, dx^0 + 2\beta_i\, dx^0\, dx^i + \gamma_{ij}\, dx^i\, dx^j, \tag{1}$$

where α is the lapse, β^i is the shift vector, and γ_{ij} is the spatial 3-metric. In this background metric, an ideal gas satisfies the following balance laws for the rest mass density D, the momentum density S_j, and the energy density E:

$$\frac{\partial}{\partial x^0}\left(\sqrt{\gamma}\begin{bmatrix} D \\ S_j \\ E \end{bmatrix}\right) + \frac{\partial}{\partial x^i}\left(\sqrt{-g}\begin{bmatrix} D\left(v^i - \frac{\beta^i}{\alpha}\right) \\ S_j\left(v^i - \frac{\beta^i}{\alpha}\right) + p\delta^i_j \\ E\left(v^i - \frac{\beta^i}{\alpha}\right) + pv^i \end{bmatrix}\right) = \sqrt{-g}\begin{bmatrix} 0 \\ T^{\mu\nu}\left(\frac{\partial g_{\nu j}}{\partial x^\mu} - \Gamma^\delta_{\nu\mu} g_{\delta j}\right) \\ \alpha\left(T^{\mu 0}\frac{\partial \log(\alpha)}{\partial x^\mu} - T^{\mu\nu}\Gamma^0_{\nu\mu}\right) \end{bmatrix}, \tag{2}$$

where γ is the determinant of γ_{ij} and g is the determinant of $g_{\mu\nu}$. In the above expression $T^{\mu\nu}$ is the energy-momentum tensor and $\Gamma^\sigma_{\nu\mu}$ are Christoffel symbols or connection coefficients; we refer the reader to [3] for more information on these terms. Finally, (ρ, v^i, p) are the primitive variables and represent the mass density, fluid 3-velocity, and fluid pressure.

2 Residual distribution schemes

In this work we make use of the residual distribution (RD) method as described in Abgrall et al. [1]. The method provides a way to extend Roe's approximate Riemann solver [6] to unstructured grids in multidimensions. The method can be used to solve general hyperbolic balance laws of the form

$$q_{,t} + \vec{\nabla} \cdot \vec{F} = \psi. \tag{3}$$

The approximate solution is stored at nodes (i.e., vertices of triangles in 2D). At every time step and in each triangle, $\mathcal{T}$, a total residual is computed,

$$\Phi^{\mathcal{T}} \approx \iint_{\mathcal{T}} \left(\vec{\nabla} \cdot \vec{F} - \psi\right)\, d\vec{x}, \tag{4}$$

which is then distributed to each node in the triangle via a multidimensional Riemann problem. We refer the reader to [1] for the details.

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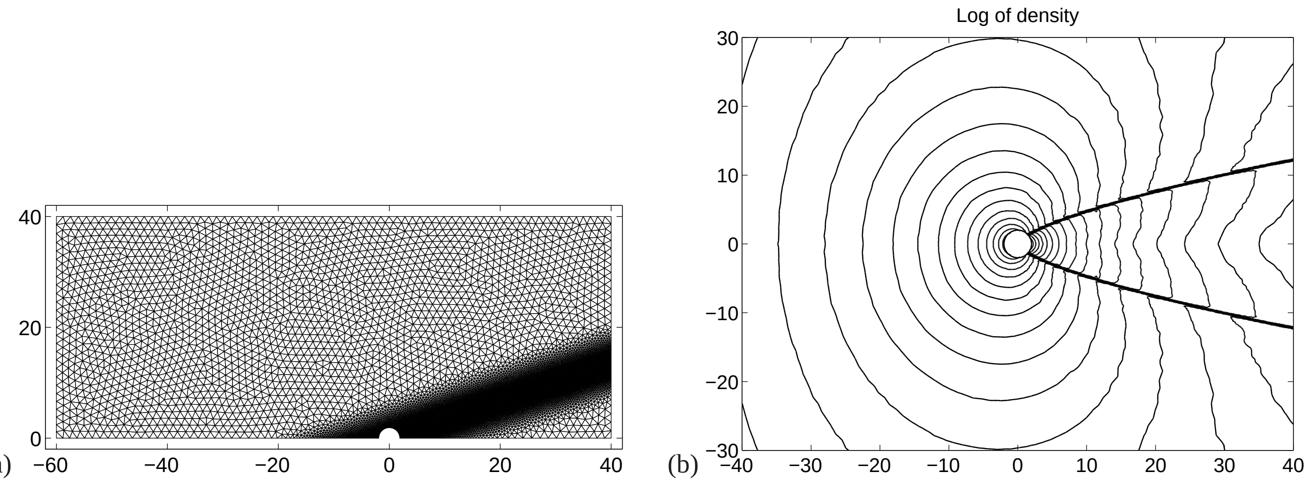

Fig. 1 (a) The computational grid used in this work. (b) Density contours for Bondi-Hoyle accretion onto Schwarzschild black hole. Supersonic gas is entering from the left boundary and impinging on a non-axisymmetric black hole centered at the origin. The gas immediately behind the black hole is subsonic and is separated from the incoming supersonic flow through a pair of symmetric shock waves. Since the initial and boundary conditions are up-down symmetric, the computation that was used to produce this figure was done only on the top half of the domain: $[-60, 40] \times [0, 40]$.

3 Bondi-hoyle accretion example

In order to test this method, we apply it the black hole accretion problem studied in Font et al. [4]. In their numerical method Font et al. use a logically Cartesian grid based on polar coordinates. In this work we instead employ an unstructured mesh and for simplicity represent velocity components in a “Cartesian” coordinate basis. The advantage of using an unstructured mesh is that we can easily adapt the grid spacing in regions near the black hole where the source terms become large and around shock waves where the order of accuracy is reduced to first order (this adaptivity would be difficult in for time accurate simulations; however, for the moment we are only interested in steady-state solutions).

The metric used in this example is the so-called Kerr-Schild metric, which can be written as

$$g_{\mu\nu} = \begin{bmatrix} -1+\frac{2}{r} & \frac{2x}{r^2} & \frac{2y}{r^2} & 0 \\ \frac{2x}{r^2} & 1+\frac{2x^2}{r^3} & \frac{2xy}{r^3} & 0 \\ \frac{2y}{r^2} & \frac{2xy}{r^3} & 1+\frac{2y^2}{r^3} & 0 \\ 0 & 0 & 0 & r^2 \end{bmatrix}, \qquad \sqrt{-g} = r, \tag{5}$$

where $r = \sqrt{x^2 + y^2}$. $r = 0$ corresponds to the center of the black hole and $r = 2$ to the event horizon. The computational grid for the example presented here is shown in Figure 1(a). An initial coarse grid calculation was done to estimate the location of the shock. From this the grid was adapted in such a way as to produce a coarse uniform resolution in regions away from the shock and a fine resolution near the shock and black hole region. The final mesh has a total of 9622 nodes and 18895 elements. Because all characteristics are inward pointing at any point inside the event horizon, the inner radius is chosen to be slightly inside the event horizon, meaning that no boundary conditions need to be specified there. We prescribe a Mach 5 inflow at the left boundary and simple extrapolation boundary conditions at the remaining boundaries. Contours of the steady state density of this flow are shown in Figure 1(b). These results are in good qualitative agreement with those of Font et al. [4], although we are able to produce higher resolution at the shocks with the unstructured grid shown in Figure 1(a).

References

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