Circle One:

Name: ________________________ 11:00-11:55 (325)

3:30-4:20 (322)

Math211-2, Fall 2007

Quiz #1: 09–07–07

No Calculators.

1. (4 Points) Suppose $f(x) = \frac{|x|}{x}$. Sketch a graph of $f(x)$ and determine the following:

$$\lim_{x\to 0^+} f(x); \qquad \lim_{x\to 0^-} f(x); \qquad \lim_{x\to 0} f(x).$$

Solution: Remember the definition of $|x|$:

$$|x| = \begin{cases} x & x \geq 0; \\ -x & x < 0. \end{cases}$$

To the left of 0, $|x| = -x$, so

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{-x}{x} = \lim_{x\to 0^-} -1 = -1.$$

To the right of 0, $|x| = x$, so

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x}{x} = \lim_{x\to 0^+} 1 = 1.$$

Both of these conditions imply the full limit, $\lim_{x\to 0} f(x)$ DNE. The most common mistake on this problem was to say $f(0) = 0$. It is important to note 0 is NOT in the domain of f!

2. (3 Points) Find $\lim_{h\to 0} \frac{(h+3)^2 - 9}{h}$.

Solution: The strategy is to work with the fraction algebraically, because if we try to plug in 0 directly, we get 0/0 which is not defined.

$$\begin{aligned} \lim_{h\to 0} \frac{(h+3)^2 - 9}{h} &= \lim_{h\to 0} \frac{(h^2 + 6h + 9) - 9}{h} = \lim_{h\to 0} \frac{h^2 + 6h}{h} \\ &= \lim_{h\to 0} \frac{h(h+6)}{h} = \lim_{h\to 0}(h+6) = 6. \end{aligned}$$

3. (3 Points) Suppose I have a continuous function f defined on the interval [-2,2]. I don't know exactly what this function is, but I do know the following values: $f(-2) = -1$, $f(-1) = 2$, $f(0) = 0$, $f(1) = -1$, $f(2) = 5$. What can you say about the number of roots (zeros) of this function?

Solution: The intermediate value theorem implies there is AT LEAST one root in $[-2, -1]$ and $[1, 2]$. We know there is one root at 0 since $f(0) = 0$. Hence, there are at least 3 roots of $f(x)$.