

Name: _____

11:00-11:55 (325)

3:30-4:20 (322)

Math211-2, Fall 2007

Quiz #2: 09-17-07

No Calculators.

1. (4 Points) Where is the following function continuous?

$$f(x) = \begin{cases} x^2 & x < 0; \\ |x - 2| & 0 \leq x \leq 4; \\ \sqrt{x} & 4 < x. \end{cases}$$

Possible points of discontinuity of f are at $x = 0$ and $x = 4$.
We need to compute the following:

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} x^2 = 0; \\ \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} |x - 2| = 2; \\ \lim_{x \rightarrow 4^-} f(x) &= \lim_{x \rightarrow 4^-} |x - 2| = 2; \\ \lim_{x \rightarrow 4^+} f(x) &= \lim_{x \rightarrow 4^+} \sqrt{x} = 2; \\ f(4) &= 2. \end{aligned}$$

All of these conditions imply $\lim_{x \rightarrow 0} f(x)$ DNE and $\lim_{x \rightarrow 4} f(x) = f(4)$ so f is discontinuous at $x = 0$ and continuous at $x = 4$. f is continuous at all other numbers.

2. (2 Points) A certain bacteria culture grows exponentially. In one hour, the population grows from 500 to 800. Write a formula expressing the population P as a function of the time t in hours.

The model is $P(t) = P_0 b^t$ where we need to determine b . $P_0 = 500$, $P(1) = 800 = 500b^1$ so $b = 8/5$ and $P(t) = 500(8/5)^t$.

3. (2 Points) Determine the following: $\lim_{n \rightarrow \infty} 12 \left(1 - \frac{1}{2n}\right)^n$. Recall $\lim_{n \rightarrow \infty} \left(1 + \frac{r}{n}\right)^n = e^r$. This limit has $r = -1/2$, so $\lim_{n \rightarrow \infty} 12 \left(1 - \frac{1}{2n}\right)^n = 12e^{-1/2}$.

4. (2 Points) Suppose $\log_3 A = b$ where A and b are positive numbers. Rewrite the given logarithm as a function of b .

$$\log_3 \left(\sqrt{3A}\right).$$

We use the product-sum rule for logarithms:

$$\log_3 \left(\sqrt{3A}\right) = \log_3 \left((3A)^{1/2}\right) = \frac{1}{2} (\log_3(3) + \log_3(A)) = \frac{1}{2} (1 + b)$$