3. (3 Points) Find the point on the graph of $y = \sqrt{x}$ that is closest to the point $(1, 0)$.

We need to minimize the distance function:

$f(x) = \sqrt{(x-1)^2 + y^2}$ from a pt on the graph to the point $(0,1)$.

$y = \sqrt{x}$ when we're on the graph, so

$$f(x) = \sqrt{(x-1)^2 + x} = \sqrt{x^2 - 2x + 1 + x} = \sqrt{x^2 - x + 1}.$$

Critical points when

$$f'(x) = 0 = \frac{1}{2\sqrt{x^2-x+1}} \cdot (x^2 - x + 1)' = \frac{2x-1}{2\sqrt{x^2-x+1}}.$$

The denominator is never zero, crit pt when $2x - 1 = 0$, or $x = \frac{1}{2}$. when $x = \frac{1}{2}$, $y = \frac{1}{\sqrt{2}}$.

To see $(\frac{1}{2}, \frac{1}{\sqrt{2}})$ really is the minimum, we need to took at the domain of $f(x)$: $[0, \infty)$.

$f(0) = 1$. $f(\frac{1}{2}) = \sqrt{(\frac{1}{2})^2 + \frac{1}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} < 1$.

$\lim_{x \to \infty} f(x) = \infty$. $\therefore$ $(\frac{1}{2}, \frac{1}{\sqrt{2}})$ really is the closest pt.

4. (Bonus) (1 Point) Is 20 minutes enough time for this quiz?

No.