

Name: Key

11:00-11:55 (325)

3:30-4:20 (333)

Math211-2, Fall 2007

Quiz #9: 12-5-07

No Calculators. Feel free to use the backside as well.

1. Suppose $\frac{dy}{dx} = xy - x$.

a. (3 Points) Find a general solution for $y(x)$.

b. (1 Point) Check your solution.

c. (2 Points) If $y(0) = 2$, find $y(x)$.

a.)

$$\frac{dy}{dx} = x(y-1), \quad \frac{dy}{y-1} = x dx, \quad \int \frac{dy}{y-1} = \int x dx$$

$$\ln|y-1| = \frac{x^2}{2} + C_0 \quad |y-1| = e^{\frac{x^2}{2} + C_0} = e^{\frac{x^2}{2}} e^{C_0}$$

new constant

$$y-1 = C e^{\frac{x^2}{2}} \quad \text{or} \quad \boxed{y = 1 + C e^{\frac{x^2}{2}}}$$

$$b.) \frac{dy}{dx} = C x e^{x^2/2} \quad xy - x = x(1 + C e^{x^2/2}) - x$$

$$= x + C x e^{x^2/2} - x$$

$$= C x e^{x^2/2} \quad \checkmark$$

~~9/8~~c) $y(0) = 2$ means $y = 2$ when $x = 0$. Plug that in:

$$2 = 1 + C e^0 = 1 + C \quad \text{so} \quad C = 1$$

$$\boxed{y(x) = 1 + e^{x^2/2}}$$

2. (4 Points Total) A drug is being intravenously fed to a patient at a constant rate of 100 milligrams per hour. At the same time, the drug is being eliminated from the patient's bloodstream at a rate that equals 10% of the amount present at any given time.

- a. (1 Point) Write a differential equation for $y(t)$, the amount in the patient's bloodstream at a time t , with t in hours and y in milligrams.
- b. (3 Points) If there is an initial amount of 50 milligrams in the patient's bloodstream, how much will there be at the end of 10 hours.

a)

$$\frac{dy}{dt} = 100 - .10y$$

b)

$$\frac{dy}{100 - .10y} = dt \quad \text{so} \quad \int \frac{dy}{100 - .10y} = \frac{-1}{.10} \ln |100 - .10y|$$

$$= \int dt = t + C_0$$

$$\Rightarrow \ln |100 - .10y| = -.10t - .10C_0$$

$$\Rightarrow |100 - .10y| = e^{-.10t} \overset{\text{new } C}{e^{-.10C_0}}$$

$$100 - .10y = C e^{-.10t}$$

$$100 - C e^{-.10t} = .10y \quad \text{or} \quad y = \frac{100}{.10} - \frac{C}{.10} e^{-.10t}$$

new C

$$y = \frac{100}{.10} + A e^{-.10t} \quad (\text{General soln.})$$

$y(0) = 50$ is the initial value, so

$$50 = 1000 + A e^0 = 1000 + A, \quad A = -950$$

$$y(t) = 1000 - 950 e^{-.10t}$$

$$y(10) = 1000 - 950 e^{-1}$$