ANSWERS to WORKSHEET 3

(1) (a) $\frac{2}{3}$ (b) 13 (c) $\frac{3}{4}$ (d) $+\infty$ (e) 0 (f) 0

(3) (a) $x_1 = 2$, $x_2 = 3$ (b) $x_1 = -3$, $x_2 = 2$ (c) $x = 4$

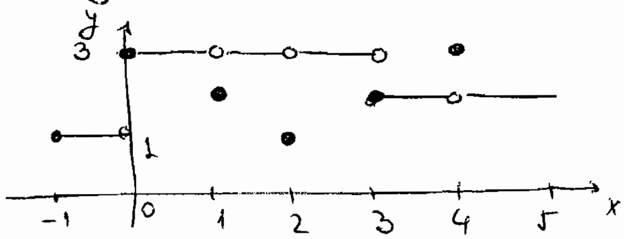
(d) $x_1 = -2$, $x_2 = \frac{1}{4}$ (e) $x_1 = 0$, $x_2 = \ln 5$

(4) (a) $1 - \frac{b}{2}$ (b) $-b$ (c) $\frac{1}{2}(1+b)$ (d) $2b$

(5) $x = 0$

(6) $c = \frac{4}{3}$

(7)



(8) $\lim_{x \to 1^+} \frac{x-1}{|x-1|} = 1$

$\lim_{x \to 1^-} \frac{x-1}{|x-1|} = -1$

(9) $t \simeq -5{,}568 \frac{\ln(0.6132)}{\ln 2}$

(10) (a) $y = 4x + 4$ (b) $x = \pm 2$ (c) No ($f'(x) > 0$ for all x)

(11) \$ 30

(12) $\frac{1}{2.108} \simeq \frac{1}{2} - \frac{1}{4} \cdot 0.108$

(13) $x = -1$ and $x = 3$

(14) (a) $\frac{dy}{dx} = \frac{(1 - \ln y)2xe^{x^2} - 2(x+1) - \frac{1}{x+1}}{2y + \frac{1}{y}e^{x^2} + 1}$

(b) $y - 1 = -\frac{3}{4}x$

(15) $\frac{dy}{dx} = -\frac{14y + e^{y^2}}{e^{y^2} 2xy + 14x}$

(16) $\frac{32}{3}\pi$ cubic feet

(17) -26 m / minute

(18) (a) $\frac{dp}{dq} = -400e^{-0.8}$ (b) $\frac{dq}{dt} = -400e^{-0.8}\frac{dp}{dt}$

(c) $\frac{dq}{dt} = -10e^{-0.8}$

(19) Critical points -2 and 0. Increasing on $(-2, 0)$. Decreasing on $(-\infty, -2)$ and $(0, \infty)$. Local min at -2. Local max at 0. Global max / min do not exist.

(20) No inflection points. Always concave up

(21) Local min at $-\frac{1}{\sqrt{2}}$, local max at $\frac{1}{\sqrt{2}}$

(22) (a) $f(-2) = -23 < 0$, $f(-1) = 5 > 0$

Since $f(-2) < 0$ and $f(-1) > 0$ (and f is continuous), $f(x) = x^5 + x + 7$ must cross the x-axis at least once between $x = -2$ and $x = -1$

(b) $f'(x) = 5x^4 + 1 > 0$ for all x

So f is always increasing, hence it cannot ~~have~~ cross the x-axis twice

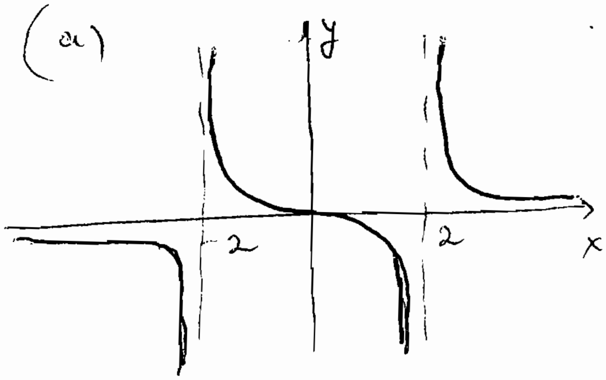
(23) Let $f(x) = e^x - x$. Then $f'(x) = e^x - 1$ and $x = 0$ is a critical point.

Since $f''(x) = e^x$, $f''(0) = 1 > 0$ so the 2nd derivative test guarantees that $x = 0$ is a local minimum. So $f(x) \geq f(0)$ for all values of x

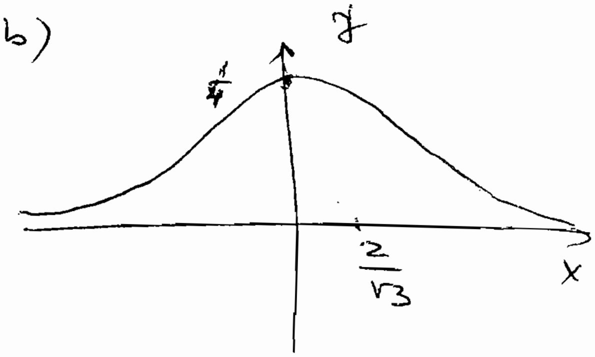
But $f(0) = 1$, so $f(x) \geq 1$ for all x.

(24) None.

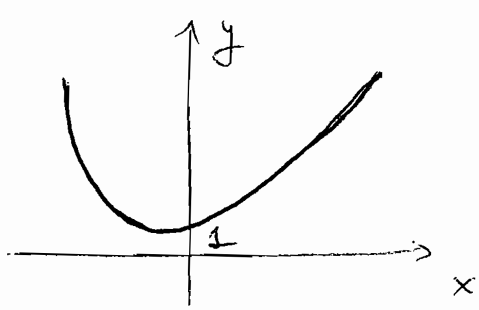
(25) (a)



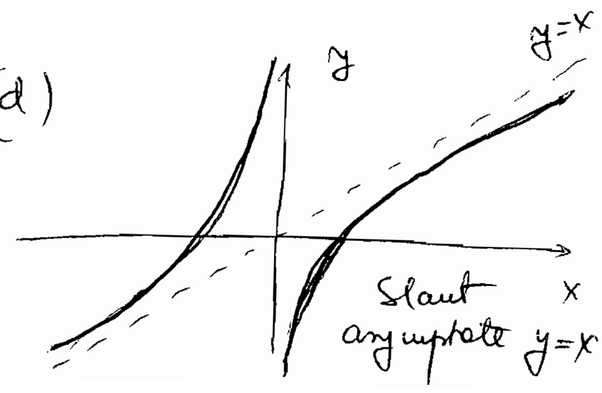
(b)



(c)



(d)



(26) (a) global min at $x=2$ with min value $f(2)=\frac{e^2}{4}$

no global max

(b) global min at $x=0$ with min value $f(0)=-1$

global max at $x=1$ with max value $f(1)=\frac{1}{3}$

(27) $(1,1)$; minimum distance $=\sqrt{2}$

(28) $y=-\frac{1}{2}x+2$ (29) $x=250$ (30) $1{,}000$

(31) 30×20 (32) 1 (33) (a) $e^x-\ln(1+e^x)+C$

(b) $x-2\ln(x+2)+C$ (c) $-\frac{1}{6}(3x^2+3x+1)^{-2}+C$

(d) $2e^{\sqrt{x}}(\sqrt{x}-1)+C$ (e) $\frac{1}{2}e^{x^2}(x^4-2x^2+2)+C$

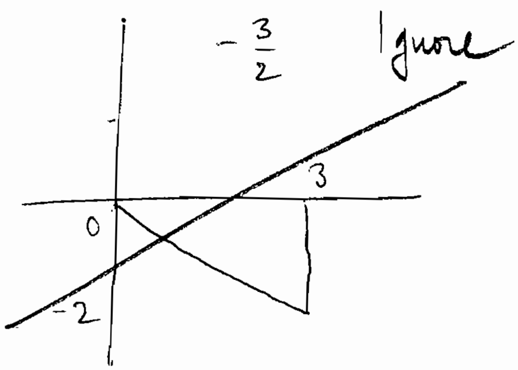
(f) $x-2\ln(1+e^x)+C$ (g) $x(\ln x)^3-3x(\ln x)^2+6x\ln x-6x+C$

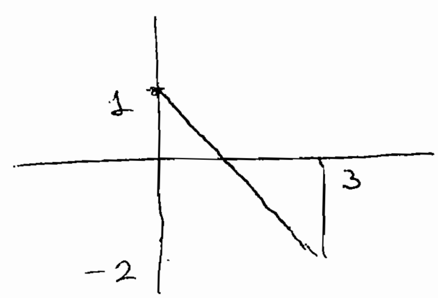
(h) $\frac{1}{2}\ln\left|\frac{e^x-3}{e^x-1}\right|+C$ (i) $\frac{4}{5}\ln|\sqrt{x}-2|+\frac{6}{5}\ln|\sqrt{x}+3|+C$

(34) $y(x)=(\ln x)^2+1$ (35) $\frac{2}{5}(e^t+2)^{5/2}-\frac{4}{3}(e^t+2)^{3/2}+\cancel{1}+\frac{2}{5}\sqrt{3}$

(36) (a) $-4\ln 2$ (b) $-\frac{1}{3}-2\ln 2$

(37)





(38) $\int_{-1}^{2}x^3\,dx=\frac{15}{4}$, total area $=\frac{17}{4}$

(39) $f(x)=1-e^{-x}$ (40) $10-e\sqrt{2}\cdot\frac{1}{\sqrt{2}}$ (41) $\frac{64}{3}$

(42) $\frac{11}{3}$ (43) 3 (44) (a) $\frac{23}{2}$ (b) 6 (c) -6

(45) $\frac{28{,}900}{3}$ (46) $CS=\frac{15}{2}$, $PS=5$ (47) $q_e=6$, $p_e=11$

$CS=110\ln\frac{5}{2}-66$

$PS=18$

48 $\dfrac{8400\left(e^{10\ln(1.2)-0.06}-1\right)}{\ln(1.2)-0.06}$

49 $192{,}500e^{2}-187{,}500$

50 $y(t)=\dfrac{e^{t}}{2-t}$

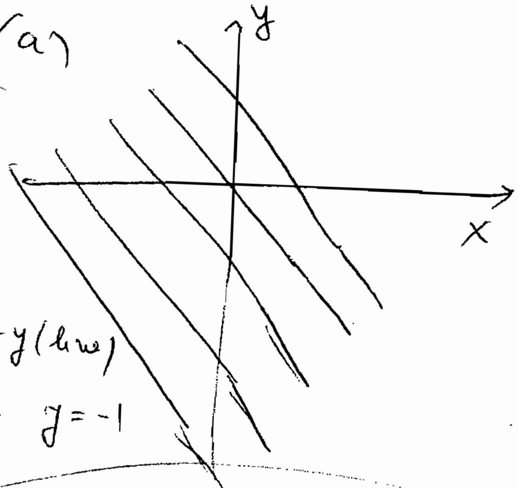
51 $M(t)=102{,}000e^{0.09t}-100{,}000$

$M(25)=102{,}000e^{2.25}-100{,}000$

52 (a) $M(0)=122{,}753$ (b) \$ 201, 247

53 $p(t)=\dfrac{5}{5-4e^{-t}}$; $\lim_{t\to 0} p(t)=1$

54 (a)



x-section at $x=0$: $z=1+y$ (line)

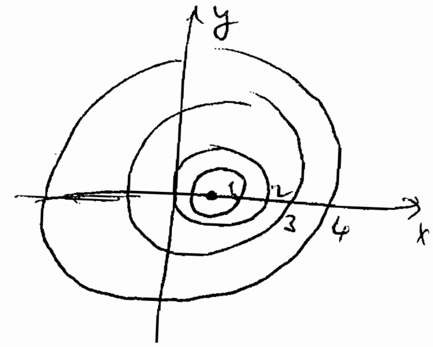
y-section at $y=-1$

$z=x$ (line)

x-section at $x=2$; $z=y^2+1$ (parabola)

y-section at $y=1$, $z=(x-1)^2+1$

(b)



55 $z=\dfrac{57}{7}x-\dfrac{13}{7}y-\dfrac{24}{7}$

56 (a) $\dfrac{\partial f}{\partial x}=2y^4e^{2xy}-5x^4$

$\dfrac{\partial f}{\partial y}=y^2e^{2xy}(3+2xy)$

(b) $\dfrac{\partial f}{\partial x}=-y^2e^{-xy^2}\ln(x^2+1)+e^{-xy^2}\cdot\dfrac{1}{x^2+1}\cdot 2x$

$\dfrac{\partial f}{\partial y}=-2xye^{-xy^2}\ln(x^2+1)$

57 (a) tg plane: $z=\ln 2$; lin aprox: $f(x,y)\approx \ln 2$

(b) tg plane: $z=16+12y$; lin aprox: $f(x,y)\approx 16+12y$

58 $D(3.012,4)-D(3,4)\approx(0.6)\cdot(0.012)$

59 (a) $(1,0)$ & $(-1,0)$ are local min & and $(0,0)$ is a saddle point

(b) $(1,2)$ & $(-1,2)$ are both local min points

60 $x=5$, $y=7$