

### PROBLEM #13, SECTION 2-3

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**#13, Section 2-3** The population of mosquitos in a certain area increases at a rate proportional to the current population, and in the absence of other factors, the population doubles each week. There are 200,000 mosquitos in the area initially, and predators (birds, bats, and so forth) eat 20,000 mosquitos/day. Determine the population of mosquitos in teh area at any time.

This problem is really two problems in one. The first problem deals with the base growth model (or radioactive decay model):

$$\begin{cases} \frac{dP}{dt} = kP \\ P(0) = P_0, \end{cases}$$

whose solution is  $P(t) = P_0e^{kt}$ . The unique thing about this function/differential equation is the concept of a half-life or doubling constant  $t_2 = 7$  days. If you sample the function at *any* time, then march forward  $t_2$  days, the population doubles (or in the case of radioactive decay, gets cut in half). This time is related to the growth rate constant  $k$  through plugging it into the function:

$$P(7) = 2P_0 = P_0e^{7k}$$

which gives  $2 = e^{7k}$ . Taking logs, and solving for  $k$ , we have  $k = \frac{\ln(2)}{7}$ . In general you would have  $k = \frac{\ln(2)}{t_2}$  which has units of 1/day or in the case of decay, we'd have  $k = \frac{\ln(0.5)}{t_{1/2}}$  which has the same units.

Note that if you mess around with exponential and log properties, you can actually rewrite this as

$$P(t) = P_0e^{kt} = P_02^{t/t_2}$$

and solve that equation for  $k$  to get the same thing we just discovered.

Anyway, the base model for our problem is

$$\begin{cases} \frac{dP}{dt} = \frac{\ln(2)}{7}P \\ P(0) = 200,000. \end{cases}$$

Now we want to add in a 'harvesting' term. Bats eat these mosquitos at a rate of 20,000 mosquitos / day. Note that the units of growth rate are  $\left[\frac{dP}{dt}\right] = \frac{\text{mosq.}}{\text{day}}$ . We can account for this additional term by adding this to our differential equation:  $\frac{dP}{dt} = \text{growth rate} - \text{death rate}$ . It becomes

$$\begin{cases} \frac{dP}{dt} = \frac{\ln(2)}{7}P - 20,000 \\ P(0) = 200,000. \end{cases}$$

You can solve this by either using the integrating factor method or separation of variables.