

Circle One:

(323) 12:05 PM - 12:55, Monday

Name: _____

(325) 02:25 PM - 03:15, Monday

(324) 12:05 PM - 12:55, Wednesday

Math319-2, Fall 2009

(326) 02:25 PM - 03:15, Wednesday

Quiz #3: 11-02-09

Due: 11-10-09.

Rules: You may discuss this quiz with me or students in the class only. Solutions must be written independently. Violating these rules constitutes a breach in the Student Code of Conduct and will not be tolerated.

1. (5 Points) If $y(x) = (a_0 + a_1x + a_2x^2 + \dots)$ is a power series with a positive radius of convergence, determine each of the following in terms of $\sum_n b_n x^n$ for some b_n . I.e. write each of the following in terms of a coefficient times x^n . Some may need to be split up into two summands.

a. y'

b. y''

c. x^2y'

d. $(x^3 + 1)y''$

e. $xy'' + (2 - x)y'$

2. (5 Points) If $y(x) = (a_0 + a_1x + a_2x^2 + \dots)$ is a power series solution centered at $x_0 = 0$ to the differential equation

$$(1 + x^2)y'' + y = 0,$$

we expect to be able to write $y(x)$ as

$$y(x) = a_0y_1(x) + a_1y_2(x)$$

since both $p(x) = 0/(1 + x^2)$ and $q(x) = 1/(1 + x^2)$ are analytic at x_0 . What is a lower bound for the radius of convergence of the power series solution $y(x)$? Find the recurrence relation for the power series solution. Find the first three terms for $y_1(x)$.

3. (5 Points) Determine the general solution to the problem

$$x^2y'' - 2xy' - 2y = 0.$$

There are two functions, y_1 and y_2 that form a fundamental set of solutions. Which function is dominant near $x_0 = 0$?

4. (5 Points) Repeat problem 3 but with the differential equation

$$x^2y'' + 7xy' + 5y = 0.$$