13.1 PROBLEM #39

$\vec{a} = 3\vec{i} - \vec{j} + \vec{k}$ so that $\vec{v}(t) = 3t\vec{i} - t\vec{j} + t\vec{k} + \vec{C_1}$ where $\vec{C_1}$ is some constant vector of integration. Plugging in $t = 0$ we see that $\vec{C_1} = \vec{v}(0)$. To find $\vec{v}(0)$ we need a magnitude and direction. We know that the length of $\vec{v}(0)$ is 2, but we also know that $\vec{v}(0)$ points in the direction of the vector between $(1, 2, 3)$ and $(4, 1, 4)$ which is $\vec{b} = 3\vec{i} - \vec{j} + \vec{k}$ because the particle travels in a straight line. We can take $\vec{v}(0) = 2\frac{\vec{b}}{|\vec{b}|}$. Integrate once more to find $\vec{r}(t)$. Plug in $t = 0$ to find the constant vector of integration.

I think the one tricky part of this problem is recognizing the fact that to find the vector $\vec{v}(0)$, we need a length and direction. They give us a length, to find the direction, we use the fact that the particle travels in a straight line.