Circle One:

Name: _____

7:45-8:35 (361)

8:50-9:40 (362)

Math222-4, Spring 2007
Quiz #1: 01–29–07
No Notes. No Calculators.
Evaluate the following integrals:

1. (*5 points*) $\int \frac{1}{1+\cos(x)}dx$ $= \int \frac{1-\cos x}{(1+\cos x)(1-\cos x)}dx = \int \frac{1-\cos x}{1-\cos^2 x}dx$

$= \int \frac{1-\cos x}{\sin^2 x}dx = \int \frac{1}{\sin^2 x}dx - \int \frac{\cos x}{\sin^2 x}dx$

$= \int \csc^2 x\,dx - \int \cot x \csc x\,dx$

$= \boxed{-\cot x + \csc x + C}$

2. (*5 points*) $\int \frac{\ln(x)\,dx}{x+x\ln^2(x)} = \int \frac{\ln x\,dx}{x} \cdot \frac{1}{1+\ln^2 x} = I$

let $u = 1 + \ln^2 x$ $du = 2\ln x \frac{1}{x}dx$

$\frac{du}{2} = \frac{\ln x}{x}dx$

$I = \int \frac{du}{2u} = \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|1+\ln^2 x| + C}$

3. (*5 points*) Derive the integration by parts formula. $\int u\,dv = uv - \int v\,du$

Start with the product rule:

$$d(uv) = u\,dv + v\,du$$

$$\Rightarrow \quad u\,dv = d(uv) - v\,du$$

Integrate both sides:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

4. (*5 points*) $\int y \ln(y)\,dy = I$ Method $dv = y\,dy$.

$u = \ln y \qquad du = \frac{1}{y}\,dy$

$dv = y\,dy \qquad v = \frac{1}{2}y^2$

$$I = \frac{1}{2}y^2 \ln|y| - \int \frac{1}{2}y^2 \frac{1}{y}\,dy = \frac{1}{2}y^2 \ln|y| - \frac{1}{2}\int y\,dy$$

$$= \boxed{\frac{1}{2}y^2 \ln|y| - \frac{1}{4}y^2 + C}$$