

Circle One:

Name: _____

7:45-8:35 (361)

8:50-9:40 (362)

Math222-4, Spring 2007

Quiz #2: 02-06-07

No Calculators.

1. (5 Points) $\int \frac{5x-2}{2x-x^2} dx$

Solution: We factor the denominator, set up the partial fractions and solve for the coefficients.

$$\frac{5x-2}{2x-x^2} = -\frac{5x-2}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}.$$

Multiply through by the denominator and plug in values for x or use the cover up method to solve for A and B :

$$-(5x-2) = A(x-2) + Bx.$$

Plugging in $x=0$ we get $A=-1$ and plugging in $x=2$ we get $B=-4$. Thus,

$$I = \int \frac{5x-2}{2x-x^2} dx = \int \left(\frac{-1}{x} + \frac{-4}{x-2} \right) dx = -\ln|x| - 4\ln|x-2| + C$$

where C is a constant.

2. (5 Points) $\int \frac{x^4}{x^2-1} dx$

Solution: For this problem we need to do long division before we can set up the partial fractions. We have:

$$\begin{array}{r} x^2+1 \\ x^2-1 \overline{) x^4} \\ \underline{-x^4+x^2} \\ x^2 \\ \underline{-x^2+1} \\ 1 \end{array}$$

The problem reduces to finding

$$I = \int \frac{x^4}{x^2-1} dx = \int \left(x^2 + 1 + \frac{1}{x^2-1} \right) dx.$$

The third term can be integrated using partial fractions:

$$\frac{1}{x^2-1} = \frac{1/2}{x-1} - \frac{-1/2}{x+1}.$$

We have

$$I = 1/3x^3 + x + 1/2(\ln|x-1| + \ln|x+1|) + C = 1/3x^3 + x + 1/2 \ln \left| \frac{x+1}{x-1} \right| + C.$$

3. (5 Points) **Solution:** $\int_0^{\pi/2} \sin^3(2x) dx$

Put

$$\begin{aligned} I &= \int_0^{\pi/2} \sin^3(2x) \, dx = \int_0^{\pi/2} (1 - \cos^2(2x)) \sin(2x) \, dx \\ &= \int_0^{\pi/2} \sin(2x) \, dx + \int_0^{\pi/2} \cos^2(2x) \sin(2x) \, dx = -1/2 \cos(2x) \Big|_0^{\pi/2} + J \\ &= -1/2(\cos(\pi) - \cos(0)) + J = 1 + J, \end{aligned}$$

where $J = \int_0^{\pi/2} \cos^2(2x) \sin(2x) \, dx$. To calculate J , make the variable sub $u = \cos(2x)$, $du = -2\sin(2x) \, dx$ into the the second equation and end up with

$$J = 1/2 \int_{u=1}^{u=-1} u^2 \, du = 1/2(1/3u^3 \Big|_1^{-1}) = 1/2(-2/3) = -1/3.$$

The end result is that $I = 2/3$.

Bonus (3 Points) Show that if $\int_0^1 f(x)f''(x) \, dx = 0$ and $f(0) = f'(1) = 0$, then $f(x) = 0$ for all x in $[0, 1]$. **Hint:** It suffices to show that $f'(x) = 0$ for all x in $[0, 1]$. Use integration by parts.

Solution: We integrate by parts as suggested. First,

$$\begin{aligned} u &= f(x) & du &= f'(x) \\ dv &= f''(x) & v &= f'(x) \end{aligned}$$

so that

$$\begin{aligned} 0 &= \int_0^1 f(x)f''(x) \, dx = f'(x)f(x) \Big|_0^1 - \int_0^1 f'(x)^2 \, dx = f'(1)f(1) - f'(0)f(0) - \int_0^1 f'(x)^2 \, dx \\ &= - \int_0^1 f'(x)^2 \, dx. \end{aligned}$$

These equations imply $\int_0^1 f'(x)^2 \, dx = 0$. Since $f'(x)^2 \geq 0$ and is continuous, we must have $f'(x)^2 = 0$ so that $f'(x) = 0$ for all x in $[0, 1]$. $f'(x) = 0$ shows that $f(x)$ is constant, but we know this constant must be zero since $f(0) = 0$.