Circle One:

Name: _____ 7:45-8:35 (361)

8:50-9:40 (362)

Math222-4, Spring 2007
Quiz #3 (Take Home): 02–16–07
Due: 02–19–07
You may discuss this quiz solely with me or other students in my discussion sessions only. Use a new sheet of paper for each problem. Write on one side only. Illegible solutions will not be graded. Do not cite an integral table as justification for your answer. All of these problems can be done using the standard methods used for homework problems.

1. (2 Points) Compute:

$$\int \frac{1}{x^{7/6}+x} dx.$$

Hint: $x^{7/6}+x = x(x^{1/6}+1)$.

2. (2 Points) Integrate:

$$\int \frac{d\theta}{\sqrt{1+\sqrt{\theta}}}.$$

3. (2 Points) Compute:

$$\int_{-1}^{3} \frac{4x^2-7}{2x+3}\, dx.$$

4. (2 Points) Integrate:

$$\int \frac{1}{(r+1)\sqrt{r^2+2r}}\, dr.$$

5. (2 Points) Compute:

$$\int_{0}^{\ln(2)} \frac{e^t}{e^{2t}+3e^t+2}\, dt.$$

6. (2 Points) The region in the first quadrant that is enclosed by the x-axis, the curve $y = 5/x\sqrt{5-x}$, and the lines $x = 1$ and $x = 4$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

7. (2 Points) Suppose $n, m \in \{1, 2, 3, \ldots\}$, the set of positive integers. Calculate $\frac{1}{\pi}\int_0^{2\pi} \sin(mx)\sin(nx)\, dx$. *Hint:* Treat the case $n = m$ as a special case.

8. (2 Points) Suppose $\int_{-\infty}^{\infty} f(x)\, dx = 1$ where $f(x) = C|x|e^{-kx^2}$ for some $k > 0$. *Hint:* For each of these parts, calculate $\int_0^{\infty} f(x)\, dx$ and $\int_{-\infty}^{0} f(x)\, dx$ separately. Add your answers. **You may assume (without justification) that** $\int_0^{\infty} e^{-x^2}\, dx = \sqrt{\pi}/2$.

(**a**) Find C.

(**b**) Find the value $\mu := \int_{-\infty}^{\infty} xf(x)\ dx$.

9. (4 Points) The **Laplace Transform** is defined as $\mathcal{L}\{f(t)\}(s) := \int_0^{\infty} f(t)e^{-st}\ dt$. It is common to write $\hat{f}(s) := \mathcal{L}\{f(t)\}(s)$.

(**a**) If $f(t) = t$, find $\hat{f}(s)$, where $s > 0$.

(**b**) If $f(t) = e^{\alpha t}$, find $\hat{f}(s)$, where $s > \alpha$.

(**c**) If $f(t) = \sin(\alpha t)$, find $\hat{f}(s)$, where $s > 0$.

(**d**) If $f(t) = \cos(\alpha t)$, find $\hat{f}(s)$, where $s > 0$.

Why must we assume these restrictions on s?
Hint: The correct answers you should expect are the following:

(**a**) $1/s^2$

(**b**) $\frac{1}{s-\alpha}$

(**c**) $\frac{\alpha}{s^2+\alpha^2}$

(**d**) $\frac{s}{s^2+\alpha^2}$

Do not be confused about the notation. For part (a), all you need to find is $\int_0^{\infty} te^{-st}\ dt$. Think of s as a fixed constant for each of the integrals you compute. Parts c and d are essentially the same.