Circle One:

Name: ______________________________ 7:45-8:35 (361)

8:50-9:40 (362)

Math222-4, Spring 2007
Quiz #4: 02–20–07
No Calculators.

1. (3 Points) Solve this differential equation explicitly for y:

$$\frac{dy}{dx} = 3x^2 e^{-y}$$

Solution:
We recognize this as a problem that can be solved by separating variables. Starting with $e^y\ dy = 3x^2\ dx$, we can integrate both sides. The left hand side is e^y, so

$$e^y = \int 3x^2\ dx = x^3 + C$$

where C is the constant of integration. Taking a logarithm of both sides gives $y = \ln(x^3 + C)$. Note that because e^y is ALWAYS positive, we must have that $(x^3 + C) > 0$, so we do not need the absolute value signs on the logarithm.

2. (3 Points) Solve this differential equation explicitly for y:

$$x\frac{dy}{dx} + 2y = x^3, \qquad x > 0, \quad y(2) = 1.$$

Solution:
First we divide by x to put it in standard form. This gives

$$\frac{dy}{dx} + \frac{2}{x}y = x^2.$$

We compute the integrating factor:

$$R = e^{\int 2/x\ dx} = e^{2\ln|x|} = x^2.$$

Multiplying this to both sides of the equation gives

$$\frac{d}{dx}(x^2 y) = x^4$$

so that $x^2 y = \int 5x^4\ dx = x^5/5 + C$, or $y = x^3/5 + 1/x^2 C$. Now we can use the initial conditions to obtain $y(2) = 8/5 + 1/4C = 1$, so $C = -12/5$. The final solution is

$$y = x^3 + \frac{-12}{5x^2}.$$