

Circle One:

Name: _____

7:45-8:35 (361)

8:50-9:40 (362)

Math222-4, Spring 2007

Quiz #9: Due 05-02-07

You may discuss this quiz with me only. DO NOT DISCUSS THIS QUIZ WITH ANYONE ELSE, EITHER INSIDE OR OUTSIDE THIS COURSE. You are allowed to use your textbook and class notes for any of the problems.

There are 9 problems.

1. (5 Points) Determine whether or not the following series converge. If it converges, find its sum.

a.) $\sum_{n=2}^{\infty} \frac{1}{(n-1)(n+1)}$.

b.) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$.

c.) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

d.) $\sum_{k=10}^{\infty} \frac{5^{k-5}}{3^{k+1}2^{k-2}}$.

e.) $\sum_{n=0}^{\infty} (\tan^{-1}(n) - \tan^{-1}(n+1))$.

2. (3 Points) This problem was taken directly from one of Professor Isaacs' old exams.

- State the 'no way' (n^{th} -term) test.
- Suppose I know that the partial sums of a series satisfy $S_n = 1 - 1/n$ for each natural number n . Does the series converge?
- What is the fourth term in the series?

3. (2 Points)

- Find the equation of a line through $(2, 3, 0)$ that is perpendicular to both vectors $\vec{u} = \vec{i} - 2\vec{j} + \vec{k}$ and $\vec{v} = 3\vec{i} + \vec{j} - 2\vec{k}$.
- Find the equation of the plane which passes through the point $(2, 3, 0)$ and is perpendicular to the line from part a.

4. (2 Points) Find the distance from the point $P(2, 2, 3)$ to the plane $2x + y + 2z = 3$.
5. (2 Points) Use vector algebra to show that squares are the only rectangles with perpendicular diagonals.
6. (2 Points) Determine if the plane $2x + 3y - 5z = 7$ is parallel, perpendicular or neither to the following planes. Justify your reasoning.
- a.) $-6x - 9y + 15z = 7$.
- b.) $4x - y + z = 10$.
7. (1 Point) Find the equation of a plane that passes through the origin and the two points $A(0, 2, 1)$ and $B(1, -3, 2)$.
8. (2 Points)
- a.) For vectors \vec{v} and \vec{u} , show that $|\vec{v} \cdot \vec{u}| \leq |\vec{v}||\vec{u}|$. This is known as the Cauchy-Schwartz inequality. Under what conditions do we have equality?
- b.) (Triangle Inequality) Suppose \vec{v} and \vec{u} are vectors. Use vector algebra to show that $|\vec{v} + \vec{u}| \leq |\vec{v}| + |\vec{u}|$. Under what conditions do we have equality? (*Hint*: Look at the quantity $|\vec{v} + \vec{u}|^2$.)
9. (1 Point) In the formula $proj_{\vec{w}}(\vec{v}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{w}|^2} \vec{w}$, the length of \vec{w} doesn't matter. Show that for $c \neq 0$, we have $proj_{c\vec{w}}(\vec{v}) = proj_{\vec{w}}(\vec{v})$